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On the relevance of the MHD approach to study the Kelvin-Helmholtz instability of the terrestrial magnetopause

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Abstract. — It is shown that the usual MHD approximation is not suitable for the description of the Kelvin-Helmholtz instability of the terrestrial magnetopause. Indeed such a description implies an ideal magnetopause with a smooth continuous variation of the plasma and field parameters occuring in an extended current sheet. However, the actual magnetopause and its adjacent plasma boundary layer are highly irregular, both spatially and temporally. Therefore, finite ion Larmor radius effects do not represent small corrections. It is argued that these small-scale inhomogeneities can change drastically the usual large-scale description of the magnetopause oscillations.

1. Introduction

Most work on the stability problem of the terrestrial magnetopause has been carried on with ideal MHD equations considering this transition as a discontinuity [see for instance Southwood, 1968]. More recent work however has taken into account of the finite thickness of the magnetopause layer [Ong and Roderick, 1972 ; Trussoni *et al.*, 1982], while retaining the MHD approximation.

It has been shown by Lerche [1966] that the growth rate for the Kelvin-Helmholtz instability of a discontinuous transition is proportional to the wave number. Therefore, for such a model, the highest growth rate occurs for the shortest wave-lengths quite inconsistently with the approximation of negligible thickness. On the other hand, it

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has been put forward [Trussoni *et al.*, 1982] that due to the finite magnetopause thickness, finite ion Larmor radius effects can be expected to be negligible. In that case, the MHD equations should remain a suitable approximation to investigate the large-scale oscillations of the magnetopause boundary.

It is the purpose of this paper to show that the MHD approximation is actually not appropriate for the study of disturbances at the terrestrial magnetopause. Such an approximation implicitly implies that the magnetopause has an infinite parallel conductivity and a vanishingly small transverse conductivity. In that case, the electric field does not strongly differ from the convection electric field. Therefore, the contributions due to the Hall effect, the pressure gradient, the temporal dependence and the effect of a finite resistivity are neglected in this MHD framework. These contributions could actually be neglected if the magnetopause were an ideal transition with smooth continuous variations occuring in an extended current sheet. However, the actual magnetopause and its adjacent plasma boundary layer are highly irregular, both spatially and temporally. Indeed, high spatial and temporal variations of plasma and field parameters inside the magnetopause layer (MPL) and plasma boundary layer (PBL) have been demonstrated by many observations [see for example Eastman and Hones, 1979]. Following Eastman, the MPL is the current layer through which the magnetic field shifts in direction. This current layer (the magnetopause) is often resolved with both plasma and field parameters and must be distinguished from the PBL which denotes the layer of magnetosheathlike plasma observed earthward of the MPL [Eastman, 1979].

From these in situ observations, it has been established that the thickness of the magnetopause layer (MPL) is much smaller than the thickness of the plasma boundary layer (PBL). Although highly variable, a typical thickness of the PBL is $\sim 0.4\ R_E$, while the magnetopause thickness ranges between 140 km and 700 km, i.e. 1-5 ion Larmor radii (R_i) of 140 km. Although the magnetic field variation takes place through the MPL, most of the plasma variations, including the velocity shear, take place inside the PBL and at its inner edge. However, scale length (L) for significant variations (greater than a factor of four) in plasma parameters within the PBL ranges between 10 km and 1000 km [Eastman, 1979]. In such a situation, it is clear that finite Larmor radius effects due to these sharp variations cannot be neglected since the ratio (L/R_i) ranges between 0.07 and 7 with the above observed values of L and of the ion Larmor radius (R_i).

Section 2 gives the basic MHD equations of an incompressible fluid. These equations are usually perturbed for the study of the Kelvin-Helmholtz instability. In section 3, it is shown that the assumption of the " frozen field " is irrelevant for both the MPL and PBL. Indeed, terms usually neglected in the generalized Ohm's law are shown to be comparable to the convection term due to the electric field. Finally, the relevance of the MHD framework to study the physical processes involved at the magnetopause surface is critically analysed in section 4.

2. Basic MHD equations

The basic equations usually adopted for the Kelvin-Helmholtz instability problem are the MHD equations of an incompressible fluid, i.e., in MKSA units [Trussoni *et al.*, 1982].

$$\frac{\partial \varrho}{\partial t} + \nabla \cdot (\varrho \mathbf{u}) = 0 \qquad (1)$$

$$\varrho \frac{\partial \mathbf{u}}{\partial t} + \varrho(\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \mathbf{J} \wedge \mathbf{B} \qquad (2)$$

$$\nabla \cdot \mathbf{u} = 0 \qquad (3)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \wedge (\mathbf{u} \wedge \mathbf{B}) \qquad (4)$$

$$\nabla \wedge \mathbf{B} = \mu_0 \mathbf{J} \qquad (5)$$

$$\nabla \cdot \mathbf{B} = 0 \qquad (6)$$

where ϱ is the mass density and $\mathbf{u}$ the bulk velocity of the plasma. p is the pressure, $\mathbf{J}$ is the current density and $\mathbf{B}$ the magnetic field. Equation (1) is the equation of continuity while equation (2) is the hydrodynamic equation of motion. Equation (3) accounts for the incompressibility of the flow. If the plasma is infinitely conducting along the field lines and infinitely resistive across them, then the magnetic field is " frozen " into the plasma and is " carried out " along at the velocity $\mathbf{u}$ as described by equation (4). Equation (5) is the Maxwell equation for the curl of $\mathbf{B}$ for which the displacement current is neglected, μ_0 being the vacuum permittivity. The divergenceless character of $\mathbf{B}$ is given by equation (6).

A perturbation technique is usually applied to these equations. Linearized perturbed equations are then used to derive a dispersion relation from which unstable modes can be deduced. In this paper however we only want to check the relevance of these basic equations for the study of the hydromagnetic stability of the terrestrial magnetopause.

The first three equations are similar in form with the original hydrodynamic equations of an incompressible fluid. However in a collisionless magnetized plasma, the pressure tensor does not remain isotropic and therefore equation (2) assumes the smallness of the anisotropy.

The MHD approximation comes from equation (4) and it is interesting to recall from which assumptions this equation is derived. To derive equation (4), it is first assumed that the electric field (**E**) is zero along the lines of force, i.e.,

$$\mathbf{E} \cdot \mathbf{B} = 0 \tag{7}$$

This implies also that the magnetic field **B** is steady or slowly varying. Secondly, it is also assumed that all the particles drift with the electric drift velocity which therefore is identical with the bulk velocity **u**.

$$\mathbf{u} = \frac{\mathbf{E} \wedge \mathbf{B}}{\mathbf{B}^2} \tag{8}$$

From (7) and (8), the electric field is

$$\mathbf{E} = -\mathbf{u} \wedge \mathbf{B} \tag{9}$$

Using the Maxwell equation

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \wedge \mathbf{E}$$

with **E** given by (9), it follows that equation (4) is reproduced. This is the usual form of the " frozen " field approximation. In this approximation, the electric drift (8) is the principal motion of the plasma. This means that gradient drifts have been left out. This is satisfactory providing

$$(\mathrm{R}_i/\mathrm{L}) \ll 1 \tag{10}$$

where R_i is the ion gyroradius and L is the scale length for spatial variations of the field (and associated plasma parameters).

3. On the relevance of the " Frozen " field approximation

The assumption (7) that the parallel electric field vanishes implies an infinite parallel conductivity ($\sigma_{/\!/} = \infty$) while the assumption (8) of the same common drift for all particles is not compatible with any transverse conductivity. In that case, the transverse Pedersen conductivity of the plasma must be zero ($\sigma_{\perp} = 0$). However, the existence of large potential drops along field lines linking the magnetopause to the polar ionosphere is undoubtely admitted from observations of precipitated electron fluxes [Arnoldy *et al.*, 1974 ; Croley *et al.*, 1978]. Therefore, the relevance of (7) is questionable at the magnetopause. On the other hand, the finite value of $\sigma_{\perp}$ cannot be neglected, not only because of the internal resistance of the medium resulting from local wave-particle interactions, but also because of the finite integrated Pedersen conductivity ($\Sigma_{\perp}$) of current systems linking the magnetopause to the ionosphere.

As shown in section 2, the " frozen " field approximation (4) is derived from the fact that the electric field reduces to the convection electric field (9). However, the actual electric field results from the generalized Ohm's law which can be written (see for instance, Seshadri, 1973) :

$$\frac{\partial \mathbf{J}}{\partial t} = \frac{e}{m_e} \nabla \cdot \boldsymbol{\psi}_e + \frac{ne^2}{m_e} (\mathbf{E} + \mathbf{u} \wedge \mathbf{B}) - \frac{e}{m_e} (\mathbf{J} \wedge \mathbf{B}) - \frac{ne^2}{m_e} \boldsymbol{\eta} \cdot \mathbf{J} \tag{11}$$

In this equation, $\boldsymbol{\psi}_e$ is the electron kinetic pressure dyad, n is the number density, e is the magnitude of the electron charge, m_e is the electron mass and $\boldsymbol{\eta}$ is the dyadic resistivity.

It is seen that terms of the generalized Ohm's law (11) neglected in the MHD approximation (9) include

$$\boldsymbol{\eta} \cdot \mathbf{J}, \quad \frac{1}{ne} \mathbf{J} \wedge \mathbf{B}, \quad \frac{1}{ne} \nabla \cdot \boldsymbol{\psi}_e \text{ and } \frac{m_e}{ne^2} \frac{\partial \mathbf{J}}{\partial t}$$

The role of the first term $\boldsymbol{\eta} \cdot \mathbf{J}$ has already be discussed at the beginning of this section. The significance of the other terms neglected in equation (9) can be evaluated in terms of the scale length L for spatial variations of the plasma and field parameters and in terms of the characteristic velocity u of the plasma [Eastman, 1979]. Considering that

$$\omega_{ce} = \frac{eB}{m_e} \tag{12}$$

$$\omega_{pe}^2 = \frac{ne^2}{m_e \varepsilon_0} \tag{13}$$

$$B \sim \mu_0 \, J \, L \tag{14}$$

(where ω_{ce} and ω_{pe} are respectively the gyrofrequency and the plasma frequency for the electrons while ε_0 and μ_0 are the vacuum permittivity and permeability respectively) we can evaluate the ratios X, Y, Z of the electric field due to convection $|\mathbf{u} \wedge \mathbf{B}|$ to the other contributions to the electric field neglected in equation (9), i.e.,

$$\left| \frac{1}{ne} \mathbf{J} \wedge \mathbf{B} \right| , \quad \left| \frac{1}{ne} \nabla \cdot \boldsymbol{\psi}_e \right| \text{ and } \left| \frac{m_e}{ne^2} \frac{\partial \mathbf{J}}{\partial t} \right|$$

respectively. It is found

$$\frac{X}{L} (m^{-1}) \sim \frac{1}{L} u \, B \frac{ne}{JB} \sim \frac{\omega_{pe}^2 u}{\omega_{ce} c^2} \tag{15}$$

where c is the velocity of light in vacuum

$$\frac{Y}{L} (m^{-1}) \sim \frac{1}{L} u \, B \frac{ne}{|\nabla \cdot \boldsymbol{\psi}_e|} \sim \frac{u \omega_{ce} m_e}{kT_e} \tag{16}$$

where kT_e is the electron thermal energy

$$\frac{Z}{L^2} (m^{-2}) \sim \frac{1}{L^2} uB \frac{ne^2}{m_e \left| \frac{\partial \mathbf{J}}{\partial t} \right|} \sim \left(\frac{\omega_{pe}}{c} \right)^2 \tag{17}$$

Taking into account of the following typical values across the MPL [Eastman, 1979]

$$n \sim 10 \text{ cm}^{-3}$$
$$B \sim 30 \, nT$$
$$u \sim 150 \text{ km/s}$$
$$T_e \sim 100 \, eV$$

for which

$$\omega_{ce} \sim 5.3 \, ks^{-1}$$
$$\omega_{pe} \sim 178 \, ks^{-1}$$

and

$$\frac{X}{L} \sim 10^{-5}\, m^{-1}$$

$$\frac{Y}{L} \sim 4.5 \times 10^{-5}\, m^{-1}$$

$$\frac{Z}{L^2} \sim 3.5 \times 10^{-7}\, m^{-2}$$

it can be seen from (15), (16) and (17) that the contributions to the electric field neglected in equation (9) become comparable (or greater) to (than) the convection electric field (i.e., $X = Y = Z \leqslant 1$) for scale lengths of the order (or smaller) of (than) 100 km for the Hall term, 22 km for the pressure gradient term and 1.7 km for the time depending term. Such scale lengths for plasma and field variations are often observed within the MPL whose mean thickness is only a few hundred kilometers. Indeed, scale lengths for significant variations (greater than a factor of four) in plasma and field parameters, as determined from hightime resolution magnetic field data, may be as small as 10 km [Eastman, 1979]. These irregularities are always present and well documented by observations with high time resolution [see also Eastman and Hones, 1979 ; Frank *et al.*, 1978]. They should not be smoothed out for convenience and cannot be neglected, even in a first approximation, because of their large amplitude. Therefore, at the magnetopause, where these irregularities are present, terms usually neglected in the generalized Ohm's law become comparable to the convection term due to the electric field. Clearly this invalidates the MHD approach.

This conclusion is even reinforced if we consider the inner edge of the PBL layer with the following typical values [Eastman, 1979]

$$n \sim 1\ \mathrm{cm}^{-3}$$
$$u \sim 50\ \mathrm{km/s}$$
$$\mathrm{B} \sim 40\ n\mathrm{T}$$
$$\mathrm{T}_e \sim 150\ e\mathrm{V}$$

for which

$$\omega_{ce} \sim 7\, k\, s^{-1}$$
$$\omega_{pe} \sim 56\, k\, s^{-1}$$

and

$$\frac{X}{L} \sim 2.5 \times 10^{-7}\, m^{-1}$$

$$\frac{Y}{L} \sim 1.3 \times 10^{-5}\, m^{-1}$$

$$\frac{Z}{L^2} \sim 3.5 \times 10^{-8}\, m^{-2}$$

Indeed, in this case the contributions to the electric field neglected in equation (9) become comparable (or greater) to (than) the convection electric field (i.e., $X = Y = Z \leqslant 1$) for scale lengths of the order (or smaller) of (than) 4000 km for the Hall term, 75 km for the pressure gradient term and 5 km for the time depending term. Remembering that typical scale lengths for significant variations in plasma and field parameters (greater than a factor of four) within the PBL ranges between 10 km and 1000 km [Eastman, 1979], it can be concluded that the MHD approximation is even much less appropriate within the PBL than it is within the MPL layer.

4. Is an MHD approach relevant to study the stability of the terrestrial magnetopause ?

In section 3, it has been shown that terms usually neglected in the generalized Ohm's law became comparable to the convection term due to the electric field for regions both inside the MPL and PBL. This is due to the presence of large amplitude irregularities whose scale lengths range between 10 km and 1000 km. At the sharp boundaries of these irregularities, finite Larmor radius effects do not represent small corrections and the inequality (10) is certainly not checked. Clearly this invalidates the MHD assumption of " frozen " field as described by equation (4). Therefore it is not relevant to use an MHD approach to study the physical processes occuring at the terrestrial magnetopause and in particular to study its stability.

Most previous work on the magnetopause stability was performed before high time resolution observations become available and this fact explains why ideal MHD was so frequently assumed in these early studies. However as it has been demonstrated in this paper, the relevance of a continuing MHD approach becomes hardly justifiable.

Actually, the small-scale inhomogeneities present inside the MPL and PBL can change drastically the usual large-scale description of magnetopause oscillations somewhat like in hydrodynamics when turbulent eddies present in an otherwise laminar flow can change the characteristics and the stability of the layer surrounding this flow.

Another point to be criticized in the MHD theory of the Kelvin-Helmholtz instability is the assumption of incompressible perturbations of the magnetopause layer. Indeed, it may be surprising that, in such a highly spatial and temporal varying layer, the plasma inside the magnetopause behaves like an ideal incompressible fluid.

It is also important to note that the layer over which the plasma bulk velocity changes significantly has a larger thickness than the layer over which the magnetic field variation takes place. As demonstrated by observations [Eastman and Hones, 1979], the plasma velocity variation does not so much take place at the magnetopause (as the magnetic field does), but the largest velocity shear occurs at the so-called plasma boundary layer (PBL). How can then the Kelvin-Helmholtz modes be unstable when the plasma velocity appears nearly unchanged across the magnetopause ?

However, a salient feature of many satellite traversals is that the magnetopause appears to be crossed repeatedly in a single pass (multiple crossings). It has therefore been suggested that the magnetopause is frequently in motion [Aubry *et al.*, 1971 ; Lepping and Burlaga, 1979]. These inferred magnetopause motions have also been interpreted as magnetopause oscillations consistent with ripples on the magnetopause surface, although deductions about the temporal structure of the magnetopause are hard to formulate from a single satellite traversal. Recently, the pair of ISEE satellites have detected some detached structures of magnetosheath-like plasma well inside the magnetopause [Eastman and Frank, 1982]. Such magnetosheath-like structures have also been detected inside the PBL by the Prognoz-7 satellite [Lundin and Aparicio, 1982]. These magnetosheath-like regions are usually associated with strong flow of solar wind ions (H^+ and He^{++}) and the presence of terrestrial O^+ ions and can be classified roughly as " newly injected " or " stagnant " [Lundin and Aparicio, 1982]. A non stationary indentation mechanism cannot explain the stagnant magnetosheath-like events with strong O^+ fluxes inside the structures. On the other hand, the theory of impulsive penetration of solar wind irregularities into the magnetosphere first proposed by Lemaire and Roth

[Lemaire and Roth, 1978] can account for these observations. It is likely that some magnetosheath-like plasma irregularities have been observed in the past during single satellite traversals but have been understood as magnetopause oscillations.

Furthermore, the role of the interplanetary magnetic field orientation with respect to the earth's dipole field is known to be important for the interaction processes between the solar wind and the magnetosphere [Sckopke *et al.*, 1976]. In an MHD framework, the importance of this factor does not appear a significant one [Trussoni *et al.*, 1982]. However, in the impulsive penetration theory of Lemaire and Roth, it is shown that the interplanetary magnetic field direction controls the capture of plasma irregularities into the magnetosphere [Lemaire *et al.*, 1979].

In conclusion, greater theoretical insight into the physical processes occuring at the terrestrial magnetopause must come from studies of the self-consistent plasma and field equations, allowing for the presence of the interplanetary magnetic field. This requires the reformulation of the problem in the kinetic domain. This is however a very difficult problem but it results from observations that there is no other alternative. Furthermore, observational evidence of large potential drops along polar-cusp field lines indicates the need for further efforts to determine the appropriate boundary conditions at the magnetopause [Willis, 1975].

In this context, a continuing MHD approach would hamper the progress to be encouraged along the line of more appropriate kinetic approaches. Indeed, it would give the false impression that the MHD framework can provide a satisfactory " Ersatz " to study the physical processes involved at the magnetopause.

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