From pointing measurements in stellar occultation to atmospheric temperature, pressure and density profiling: simulations and first GOMOS results.

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***Abstract*— In this work we consider the problem of determination of the stratospheric density and temperature profiles from the refractive-angle measurements. This study introduces new geophysical products, which can be obtained from the GOMOS instrument located on board the Envisat satellite. The procedure of temperature reconstruction consists of the following steps. First, the refractivity is reconstructed from the refractive angle measurements using the inversion of Abel-type integral. Refractivity is connected with air density via the Edlen formula. The pressure profile is calculated then using the hydrostatic equation. Finally, the temperature profile is determined from density and pressure data using the state equation of an ideal gas.**

The realistic error analysis was performed. Sensitivity of the inversion procedure to the main error sources is studied. It is shown that both the sampling frequency and instrumental noise are very important for the reconstruction while the influence of scintillation effect and chromatic smoothing are negligible.

The first results of density and temperature retrieval from the GOMOS measurements are presented.

I. INTRODUCTION: REFRACTIVE ANGLE MEASUREMENTS WITH STELLAR OCCULTATION INSTRUMENTS

Stellar occultation instruments (such as GOMOS flying on board ENVISAT and COALA being developed now by ESA) must have high pointing accuracy, because they need to follow point sources of light. This allows accurate measurements of the refractive angle in the limb viewing geometry.

Stellar occultation measurements cover altitude range of 120-10 km with the reference point at the altitude about 120 km. The pointing system includes the steering front mechanism and the star tracker. The GOMOS and COALA star trackers operate at sampling frequency 100Hz in NIR wavelength range (GOMOS 625-950 nm; COALA 675-775 nm).

Instrumental noise contains random and harmonic components. Taking into account self-calibrating features of the stellar occultation measurements, the errors due to misalignment in assembly and the errors due to alignment variations having time period much longer than the occultation duration (e.g. thermal effects) can be eliminated. The different contributions to the error budget can be listed as follows:

- Error due to platform attitude instability,
- Error in pointing mirror attitude restitution,
- Errors due to structural vibration,
- Errors in the determination of the actual position of the star image on the sensor.

Table I shows typical values of the instrumental error contributions. The harmonic components can be filtered out from the signal, so that the main contributor is the random instrumental noise. The last row of the table shows the limit pointing accuracy values for the GOMOS and COALA instruments in the case when the harmonic components are filtered out and the random components are smoothed down to sampling frequency 2 Hz.

II. FORWARD MODEL AND INVERSION ALGORITHM

Forward model consists of determination of the refraction angle from known density profile of the atmosphere. We assume that the refractive index n depends on the wavelength λ of the light and on the neutral air density ρ according to Edlen formula [1]:

$$n = 1 + C(\lambda)\frac{\rho}{\rho_0}, \tag{1}$$

where ρ_0 is the air density at the Earth surface and $C(\lambda)$ is a constant depending on the wavelength λ as:

$$C(\lambda) = 10^{-6}\left(83.42 + \frac{24060}{130 - \lambda^{-2}} + \frac{160}{39 - \lambda^{-2}}\right) \tag{2}$$

where λ is in micrometers.

Under the spherical symmetry assumption, refractive angle ϵ may be determined as

$$\epsilon = -2\int_{r_t}^{\infty} \frac{d(\ln n)}{dr}\frac{adr}{\sqrt{n^2r^2 - a^2}} \tag{3}$$

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TABLE I

ERROR BUDGET OF POINTING ACCURACY

Contributor	GOMOS	COALA
harmonic		
platform attitude instability	$6\mu rad@1Hz$	5-$50\mu rad@1-50Hz$
structural vibration	$3\mu rad@88Hz$ $30\mu rad@150Hz$	10-15 μrad
random		
pointing mirror	$3\mu rad@10Hz$	5-10 $\mu rad@100Hz$
star sensor	15 $\mu rad@100Hz$	10-20 $\mu rad@100Hz$
Total (smoothed down to 2 Hz)	2.75 μrad	1-2 μrad

The spherical symmetry of the atmosphere is the general assumption of the forward model. Ignoring the effect of horizontal gradients of refractive index at right angles to the direction of light propagation introduces the error, which is estimated to be below 1% for altitudes above 10 km (e.g. [5], [6]).

Applying the inverse Abel transform, the refractive index we can obtain from the reconstruction formula as

$$n(y) = \exp\left(\frac{1}{\pi}\int_y^\infty \frac{\epsilon(a)da}{\sqrt{a^2-y^2}}\right), \tag{4}$$

where $y = n(r)r$ is the refractive altitude. Real geometric altitudes can be determined as $r = \frac{y}{n(y)} - R$.

The density profile $\rho(r)$ can be obtained from the refractivity profile using Edlen's formula. We can calculate then pressure P at the altitude r using the hydrostatic equation.

Finally, the temperature can be determined from the state equation of ideal gas.

$$T(r) = k\frac{P(r)}{\rho(r)} \tag{5}$$

Statistical optimization can significantly improve the accuracy of the retrieval ([7], [8]). For stellar occultation, it is convenient to optimize the density retrieval. The optimal estimation ρ_{opt} is the weighted combination of the the a-priori profile ρ_a and the reconstructed profile ρ

$$\rho_{opt} = \rho_a + (C_\rho^{-1} + C_a^{-1})^{-1}C_\rho^{-1}(\rho - \rho_a) \tag{6}$$

The optimization affects the retrieval mainly in the upper atmosphere, where signal-to-noise ratio is low, while in the lower atmosphere the retrieval is determined by observation data.

III. ERROR SOURCES

The most important factors, limiting the accuracy of retrieval, are: limited sampling frequency of measurements, instrumental error, chromatic smoothing and scintillation. In our simulations we assume that the instrumental noise is normally distributed. We also assume that the harmonic perturbations are filtered out from the measurements data.

Fig. 1a shows the total observation error for GOMOS measurements constructed by adding the covariance matrices from the instrumental noise (represented at 2 Hz sampling frequency), forward modeling, chromatic smoothing and scintillation.

The dominant error source for altitudes below 20 km is the forward modeling error, while the instrumental error dominates in the upper atmosphere. The extensive study of influence of these error sources and sampling frequency of the measurements on accuracy of the reconstruction was performed [9]. Both the sampling frequency and instrumental noise are very important for the reconstruction while the influence of scintillation effect and chromatic smoothing are small.

The estimated accuracy of temperature profiling with GOMOS and COALA instruments are presented in Fig. 1b. The simulations with all error sources included were performed for the real GOMOS occultation geometry. The error of the reconstruction was estimated applying linearization of the problem as proposed in [7]. The instrumental noise is taken 2.75 μrad at 2 Hz sampling frequency for the GOMOS instrument and 1 μrad for the COALA instrument. Application of statistical optimization leads to significant improvement of performance at a price of degraded vertical resolution with altitude. This error analysis is consistent with results obtained by Monte Carlo simulations [9].

Error of the reconstruction grows very rapidly with altitude: this is the consequence of rapid decay of signal-to-noise ratio with altitude. In the present design, it is possible to measure temperature profile within accuracy 2 K up to 25 km with GOMOS instrument and up to 35 km with COALA instrument.

IV. FIRST GOMOS RESULTS

The GOMOS refractive angles can be retrieved from the pointing data (azimuth and elevation angles) of Steering Front Assembly (SFA) and star tracker (SATU). The sampling frequency of SFA and SATU recordings are 10 Hz and 100 Hz, respectively. This gives resolution better than 300 m in the lower atmosphere. In the simulations we assumed that std of measurement error does not depent on altitude. However, the evident multiplicative structure of the noise can be observed in SATU data (Figure 2). The SFA pointing angles error contains not only the random, but also systematic components. At the moment, the pointing data are in the validation phase. Comparison of the first reconstructed temperature and density

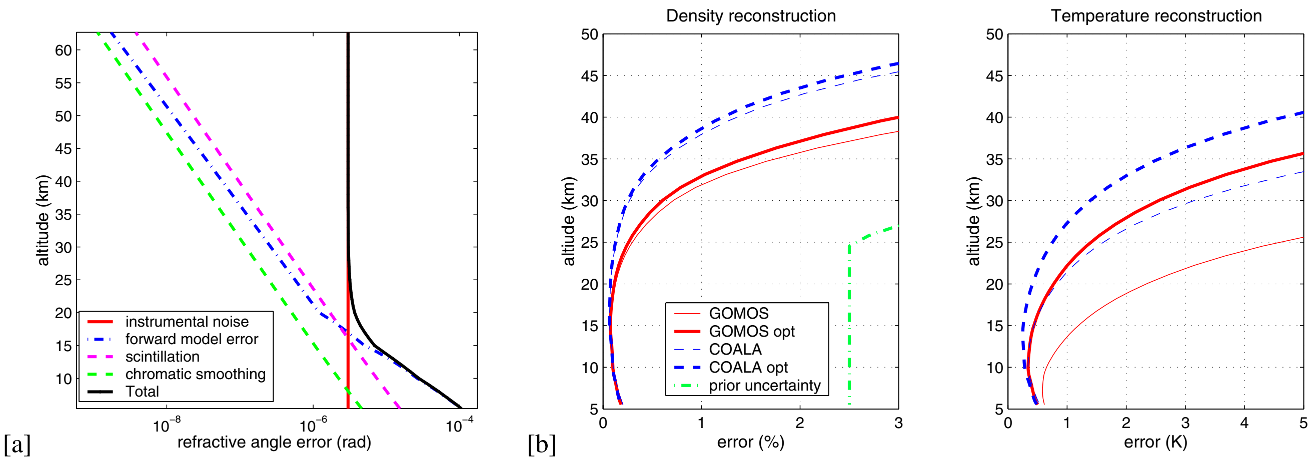

Fig. 1. [a] - Total measurement error budget for GOMOS measurements. The instrumental noise is represented at 2 Hz sampling frequency; [b]- Estimated accuracy of the reconstruction for GOMOS(solid lines) and COALA (dashed lines) instruments. Inversion algorithms: statistical optimization(bold lines) and inversion without optimization (light lines)

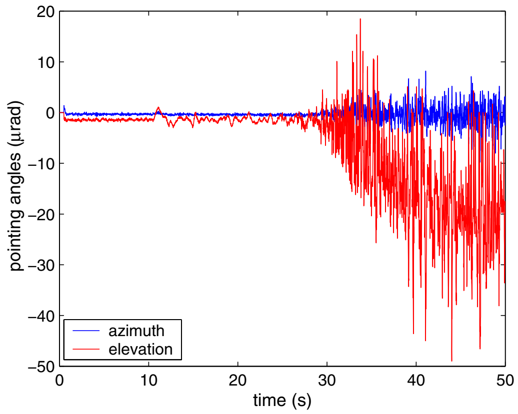

Fig. 2. SATU azimuth and elevation angles for occultation R02178/S001.

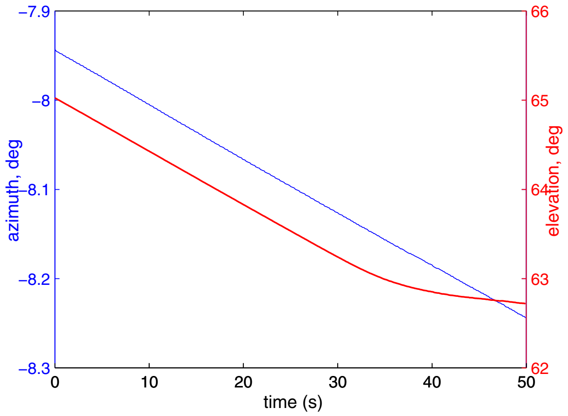

Fig. 3. SFA azimuth and elevation angles for occultation R02178/S001.

profiles with ECMWF data show a deviation, which is slightly larger than it was predicted in simulations.

V. CONCLUSIONS

In this work we have studied possibility of atmospheric density, pressure and temperature reconstruction from the refractive angle measurements in stellar occultation. The inversion algorithm with statistical optimization of the density reconstruction was discussed. Realistic error analysis was performed. It is shown that both the sampling frequency and instrumental noise are very important for the reconstruction while the influence of the scintillation effect and chromatic smoothing is small.

The simulations have shown that it is possible to measure temperature profile with accuracy 2 K up to 25 km with GOMOS instrument, and up to 35 km with COALA instrument.

However, structure of the instrumental noise should be revised using the GOMOS data. Preliminary retrieval results from the GOMOS data show, that after solving instumental problems, density and temperature profiles can be accurately reconstructed from the pointing measurements.

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