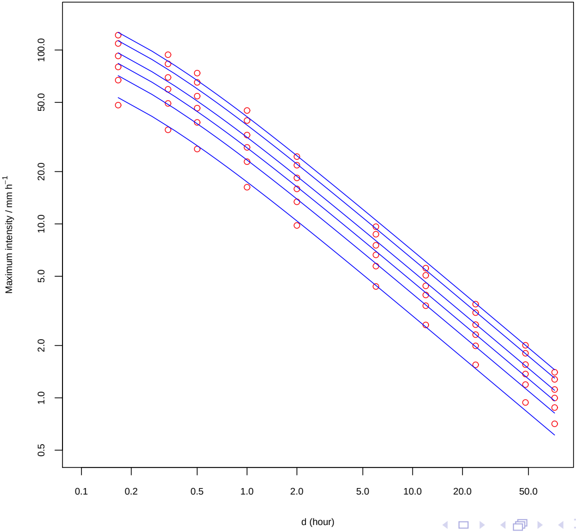
A Multiscaling Model of an Intensity-Duration-Frequency Relationship for Extreme Precipitation

Hans Van de Vyver
Royal Meteorological Institute of Belgium

September 21, 2017

IDF-curves (Uccle, Belgium)



General IDF relationship

General IDF relationship (Koutsoyiannis, Kozonis & Manetas, 1998):

- T-year return level of rainfall intensity:

$$i_T(d) = \frac{a(T)}{(d+\theta)^\eta}, \qquad \text{with} \qquad \theta > 0, \qquad 0 < \eta < 1,$$

with

$$a(T) = F_Y^{-1}(1 - 1/T),$$

and $F_Y(y)$ is the CDF of the scaled intensity, $I(d)\, b(d)$.

Generalized extreme value (GEV) distribution

- Let $X_1, \ldots, X_m$ be a sequence of iid random variables.
- **Block maxima** $M_m = \max\{X_1, \ldots, X_m\}$.
- CPF: $P\{M_m \leq z\}$.
- For sufficiently large m, $P\{M_m \leq z\}$ is approximated by the **generalized extreme value (GEV)** distribution

$$G(z; \mu, \sigma, \gamma) = \exp\left[-\left(1 + \gamma\,\frac{z-\mu}{\sigma}\right)^{-1/\gamma}\right].$$

- In practice: blocks of one year.

Generalized extreme value (GEV) distribution

- **Location parameter** μ specifies center of distribution.
- **Scale parameter** σ determines size of deviations.
- **Shape parameter** γ determines rate of tail decay.
- **Return period**

$$T = \frac{1}{1 - G(z)}.$$

- **Return level**

$$z(T) = \mu - \frac{\sigma}{\gamma}\left\{1 - \left[-\log\left(1 - \frac{1}{T}\right)\right]^{-\gamma}\right\}.$$

Simple scaling models

- **Series of d-hourly intensity:** $I_1(d)$, $I_2(d)$,...
- **Annual maximum intensity:**

$$I(d) = \max\{I_1(d), I_2(d), \ldots, I_{N_y}(d)\}.$$

- **Simple scaling.**
  - *Strict sense:*

$$I(\lambda\, d) = \lambda^{-\eta}\, I(d), \qquad \text{with } 0 < \eta < 1.$$

  - *Wide sense:*

$$E[I^q(\lambda\, d)] = \lambda^{-q\,\eta}\, E[I^q(d)].$$

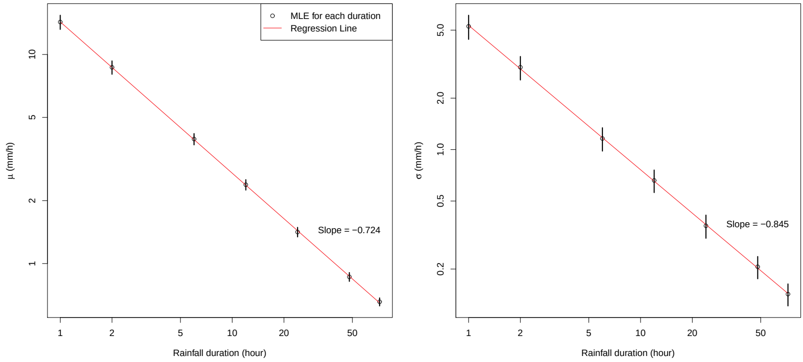
- **Multiscaling:**

$$E[I^q(\lambda\, d)] = \lambda^{-\alpha_q}\, E[I^q(d)], \qquad \text{with } \alpha_q = q\,\varphi_q\,\eta.$$

Scaling GEV-models

- We assume
$$I(d) \sim \mathrm{GEV}[\mu(d), \sigma(d), \gamma].$$
- **Simple scaling hypothesis** *(Menabde et al., 1999; Nguyen et al., 1998)*:
$$\mu(d) = d^{-\eta}\,\mu, \qquad \sigma(d) = d^{-\eta}\sigma, \qquad \gamma(d) = \gamma,$$
with $0 < \eta < 1$. $\Rightarrow$ Equivalent with the general relationship.
- **Multiscaling hypothesis:**
$$\mu(d) = d^{-\eta_1}\,\mu, \qquad \sigma(d) = d^{-\eta_2}\sigma, \qquad \gamma(d) = \gamma,$$
with $0 < \eta_1 \leq \eta_2$, and $\eta_1 < 1$.
$\Rightarrow$ Not covered by the general relationship.

Illustration: Uccle precipitation



- Location parameter: **slope = -0.724.**
- Scale parameter: **slope = -0.845.**

Bayesian inference

- N year of IDF-data:

$$\mathbf{i} = \underbrace{\begin{pmatrix} i_{11} & \cdots & i_{1M} \\ \vdots & \ddots & \vdots \\ i_{N1} & \cdots & i_{NM} \end{pmatrix}}_{M \text{ durations}}.$$

- Density function of IDF-data $\mathbf{i}$ is $L(\mathbf{i};\psi) = L(\mathbf{i}\,|\,\psi)$.
- Bayesian rule

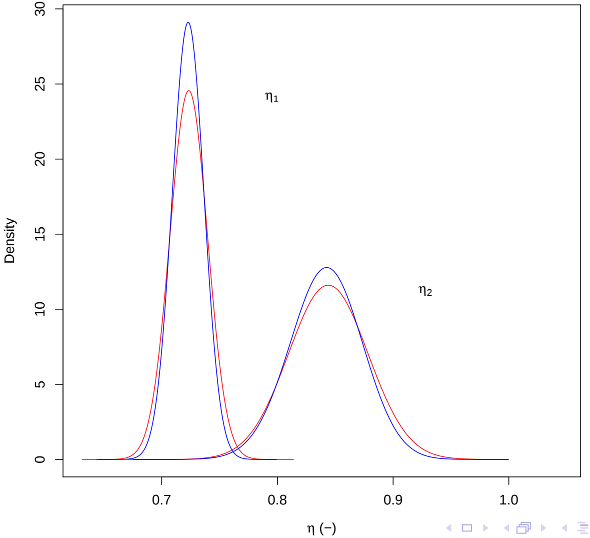
$$\pi(\psi\,|\,\mathbf{i}) = \frac{L(\mathbf{i}\,|\,\psi)\,\pi(\psi)}{\int_\Psi L(\mathbf{i}\,|\,\psi)\,\pi(\psi)d\psi},$$

with

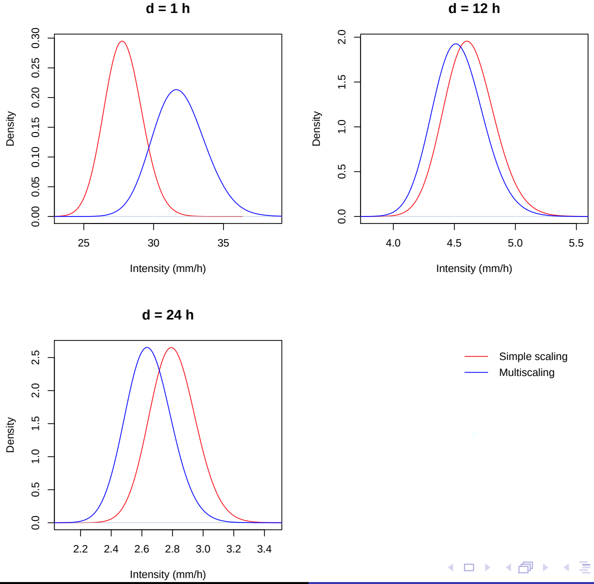
  - $\pi(\psi)$: **prior distribution**.
  - $\pi(\psi\,|\,\mathbf{i})$: **posterior distribution**.

- Van de Vyver, H. (2015) *Bayesian estimation of Intensity-Duration-Frequency relationships.* J. Hydrol. **529**, 1451–1463.

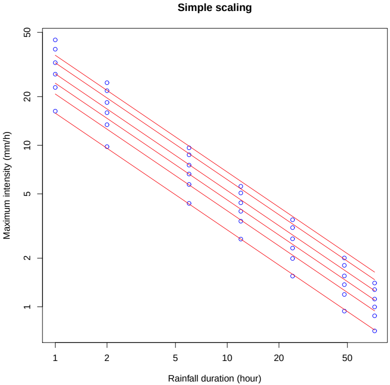
Bayesian inference: scaling exponents

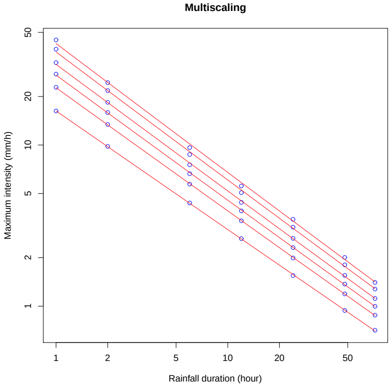


Bayesian inference: 20-year return levels

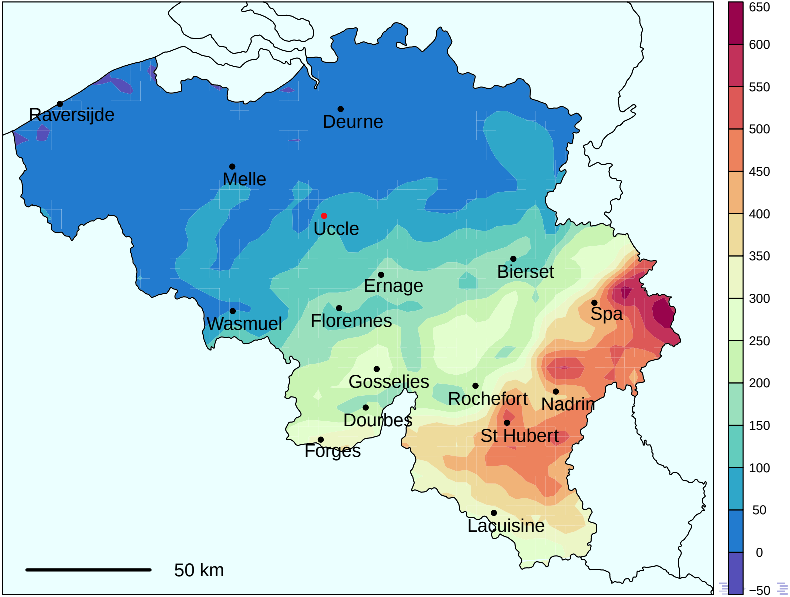


IDF-curves (Uccle, Belgium)





Station location



Model selection

	Simple scaling			Multiscaling			
Station	$\bar{\eta}$	AIC	BIC	$\bar{\eta_1}$	$\bar{\eta_2}$	AIC	BIC
Rochefort	0.712	681.1	686.8	0.718	0.831	671.6	678.9
Forges	0.702	812.8	818.4	0.720	0.878	792.3	799.6
Dourbes	0.691	656.2	662.0	0.702	0.810	648.9	655.9
Nadrin	0.691	753.0	758.9	0.695	0.832	734.6	741.9
Uccle	0.723	2182.5	2193.0	0.723	0.844	2138.2	2151.3
Melle	0.717	691.9	697.9	0.715	0.795	689.6	696.4
Lacuisine	0.637	844.6	850.5	0.643	0.791	825.8	833.2
Wasmuel	0.726	608.9	614.7	0.737	0.876	594.5	601.8
Ernage	0.730	559.9	565.4	0.729	0.852	551.1	557.9
St Hubert	0.653	693.9	699.7	0.656	0.792	675.0	682.4
Bierset	0.747	873.8	879.7	0.750	0.846	867.1	874.2
Spa	0.666	793.2	799.0	0.669	0.757	788.8	795.8
Deurne	0.713	729.6	735.5	0.711	0.799	724.5	731.4
Gosselies	0.708	624.7	630.0	0.715	0.799	621.4	627.8
Florennes	0.727	751.8	757.5	0.724	0.824	746.2	753.2
Raversijde	0.707	768.8	774.7	0.711	0.794	764.3	771.4

Conclusions

- **Simple scaling GEV-model:**

$$\mu(d) \sim d^{-\eta}, \qquad \sigma(d) \sim d^{-\eta}.$$

$\Rightarrow$ Parameters scale with one common exponent.

- **Multiscaling GEV-model:**

$$\mu(d) \sim d^{-\eta_1}, \qquad \sigma(d) \sim d^{-\eta_2}.$$

$\Rightarrow$ Parameters scale with two different exponents.

- **Bayesian inference** of scaling models.
  Advantages:
  - Assessment of uncertainty.
  - Model comparison with information crition (AIC, BIC).
- **Multiscaling is significantly better than simple scaling!**
- **Open question:** *IDF characteristics worldwide?*

Thank you for your attention!