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Modeling Atmospheric Loss by the Discrete Velocity Method

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Abstract

The work presented here is mainly based on the paper: *Discrete velocity model for an escaping single-component atmosphere* by William J. Merryfield and Bernie D. Shizgal [3]. We have reproduced their model in order to contrast it with Direct Simulation Monte Carlo models as well as to apply it to different planetary bodies' atmospheres. Also, we wish to show advantages of using discrete velocity method, such as computation time gain, numerical error reduction and scientific relevance regarding planetary atmospheres.

1. Introduction

Understanding the evolution of an atmosphere along with its composition is of crucial interest in planetary science. It can allow us to comprehend the current state of some planetary bodies' atmospheres as well as to study its evolution. One important process affecting the atmosphere is atmospheric loss. There are two types of atmospheric loss mechanisms, thermal and non-thermal escape. In the former we can find two extreme cases, one is the Jeans or evaporative escape and the other one is the hydrodynamic escape. We focus our work on thermal escape and will compare it to the Jeans escape, which is a representative case for most planetary atmospheres.

To correctly model atmospheric escape, one must model the upper atmosphere and exosphere, including the transition from collisional to collisionless regimes. In the last decades the most commonly used method has been the Direct Simulation Monte Carlo (DSMC) [2, 4]. This method suffers from statistical noise and becomes slow when simulating denser and lower parts of atmospheres. These reasons are mainly why we wished to apply the Discrete Velocity Method, as in [3], and try to find possible improvements in order to have a more efficient method.

2. Discrete velocity method

The Discrete Velocity Method (DVM) is a numerical technique often used in rarefied gas dynamics to solve the Boltzmann equation. The technique discretizes the velocity space and solves the PDE using finite difference schemes. The number of equations in the system is product of the number of spatial and velocity grid points; this means that the number of equations to be solved can become quite large. Typically, the most time consuming operations are those performed to compute the collisional integral of the Boltzmann equation described below. Because we are using finite difference, the error in our solution can be characterized using standard numerical error analysis, as opposed to DSMC which can be dominated by statistical noise.

3. Atmospheric escape process

To solve the problem of escaping single-component atmosphere we need to solve the nonlinear Boltzmann equation. Assuming spherical symmetry, we consider the radial dependence and cartesian coordinates for the velocity space, this equation can take the following:

$$\frac{\partial f_i}{\partial t} + v_z \frac{\partial f_i}{\partial r} - \frac{1}{r}\left[v_x v_z \frac{\partial f_i}{\partial v_x} + v_y v_z \frac{\partial f_i}{\partial v_y} - (v_x^2 + v_y^2)\frac{\partial f_i}{\partial v_z}\right] - \frac{GM}{r^2}\frac{\partial f_i}{\partial v_z} = C[f_i] \quad (1)$$

where C is the collision term restricted to binary collisions and whose discretized form is given as:

$$C[f_i] = \sum_{j=1}^{p} \sum_{k,l} A_{ij}^{kl} (f_k f_l - f_i f_j)/\Delta v^3 \quad (2)$$

The sum over j account for all the possible collisions between pairs of particles formed with particles of velocity vector $\vec{v_j}$ and $\vec{v_i}$ in the discrete and finite velocity space. The p variable is for the number of velocity vectors in the 3D velocity space. The sum over k and l is for all the outcomes formed after a particular collision that respect the conservation of momentum and kinetic energy. A takes into account the collisional cross section, relative velocity and total number of outcomes.

Using combination of numerical finite difference schemes we solve for $f_i(t, \vec{v}, r)$, which is the velocity distribution of particles.

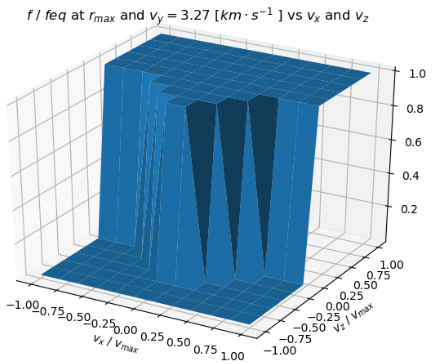

Figure 1: Plot of the distribution scaled to the Maxwell distribution versus v_z and v_x at r_{max} after 16400s for a free molecular flow model. The model used an unequal grid in r with forty points, an equal grid in v with twelve points in each component of velocity.

In Figure 1 we plot the results of a simulation of free molecular flow scaled to the Maxwell distribution. The depletion area is due to escaping particles, which we enforce as an upper boundary condition. From this type of solutions we are able to compute a series of macroscopic variables such as number density, temperature, bulk velocity components and the heat flux that characterize the atmosphere. The escape flux is calculated also as follows:

$$F_{Escape} = \sum_i v_i f(t, v_i, r)(\Delta v)^3 \qquad (3)$$

where i means particles having a velocity magnitude equal or greater than the escape velocity and directed upwards.

4. Summary and Conclusions

We apply DVM to a simple case of atmospheric escape to obtain the velocity distribution in the upper atmosphere. This method correctly models the transition from the collisional to collisionless regimes of the atmosphere. We compare our solution to the more standard DSMC method. We also compare the performance of the models, both the errors and computational effort. The promising results encourage us to foresee more complex situations of the model, for example including multiple species or photochemistry.

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