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Model of imbalanced kinetic Alfvén turbulence with energy exchange between dominant and subdominant components

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ABSTRACT

Alfvénic turbulence in the fast solar wind is imbalanced: the energy of the (dominant) waves propagating outward from the Sun is much larger than energy of inward-propagating (subdominant) waves. At large scales Alfvén waves are non-dispersive and turbulence is driven by non-linear interactions of counter-propagating waves. Contrary to this, at kinetic scales Alfvén waves become dispersive and non-linear interactions become possible among co-propagating waves as well. The study of the transition between these two regimes of Alfvénic turbulence is important for understanding of complicated dynamics of imbalanced Alfvénic turbulence. In this paper, we present a semiphenomenological model of the imbalanced Alfvénic turbulence accounting for the energy exchange between the dominant and subdominant wave fractions. The energy transfer becomes non-negligible at sufficiently small yet still larger than the ion gyroradius scales and is driven by the non-linear beatings between dispersive dominant(subdominant) waves pumping energy into the subdominant(dominant) component. Our results demonstrate that the turbulence imbalance should decrease significantly in the weakly dispersive wavenumber range.

Key words: turbulence – solar wind.

1 INTRODUCTION

It is long known that large-scale turbulent fluctuations in the solar wind are dominated by the Alfvén waves (AWs; Belcher & Davis 1971). Since the AW represents an exact solution of the incompressible ideal magnetohydrodynamics (MHD), the energy cascade in the AW turbulence is governed by non-linear interactions among counter-propagating waves (Iroshnikov 1963; Kraichnan 1965). Theory of the Alfvénic turbulence developed by Goldreich & Sridhar (1995) predicts that the turbulence is anisotropic, dominated by perpendicular cascade. With increasing $k_\perp$ the turbulent fluctuations become highly anisotropic, $k_\perp \gg k_\shortparallel$, and establish the Kolmogorov-like energy spectrum $E(k_\perp) \sim k_\perp^{-5/3}$, where $k_\perp$ and $k_\shortparallel$ are the perpendicular and parallel wavenumbers with respect to the background magnetic field $\boldsymbol{B}_0$. Anisotropy of the MHD turbulence was observed *in situ* in the solar wind (Horbury, Forman & Oughton 2008) and reproduced in numerical simulations (Cho & Vishniac 2000).

Alfvénic turbulence in the fast solar wind is strongly imbalanced in the sense that amplitudes of the waves propagating from the Sun are significantly larger than amplitudes of sunward propagating waves (Tu, Marsch & Thieme 1989). Imbalanced MHD turbulence became recently the subject of extensive theoretical and numerical studies. In the context of strong turbulence, several phenomenological models have been proposed (Lithwick, Goldreich & Sridhar 2007; Beresnyak & Lazarian 2008; Candran 2008; Gogoberidze & Voitenko 2016) but no real consensus has been reached. Numerical simulations of the imbalanced MHD turbulence lead to controversial results, which is partly caused by the fact that the strong imbalance reduces the strength of non-linear interactions thus requiring both the longer integration times and the higher spatial resolutions (Beresnyak & Lazarian 2009; Perez & Boldyrev 2009).

When the perpendicular lengths scale of the turbulent perturbations approach the ion gyroradius $\rho_{\rm i}$ AWs transform into kinetic Alfvén waves (KAWs; Hasegava & Chen 1976). Non-linear interactions of KAWs differ significantly from non-linear interactions of AWs. One of the main differences is the fact that for KAWs, contrary to the MHD AWs, the non-linear interactions are possible not only among counter-propagating but also among co-propagating waves. Kolmogorov-like self-similarity analysis predicts that in the inertial interval of the KAW turbulence the energy spectrum should scale as $E(k_\perp) \sim k_\perp^{-7/3}$. Recent observations of small-scale magnetic perturbations in the solar wind confirm that at scales comparable to ρ_i the turbulent spectrum becomes significantly steeper than in the MHD range, its spectral index varies between −2 and −4 (Leamon et al. 1999; Bruno, Trenchi & Telloni 2014; Roberts, Li & Jeska 2015).

Recently, Voitenko & De Keyser (2011, 2016) developed a semiphenomenological model of the imbalanced Alfvénic turbulence covering both the MHD and kinetic range. They found that in the strongly imbalanced case the transition from the MHD to kinetic turbulence occurs through an intermediate interval where the energy transfer is dominated by non-linear interactions among co-propagating dominant waves. In particular, Voitenko & De Keyser (2016) have shown that in the weakly dispersive intermediate interval

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the energy spectral slope can be as steep as −3 and this interval will be followed by the strongly dispersive interval with the spectral slope −7/3. These results were obtained in the assumption that the energy exchange between dominant and subdominant components is negligible and the spectral fluxes within these components are conserved.

However, numerical simulations of the reduced electron MHD equations, which are suitable to study KAWs in the strongly dispersive range (Boldyrev et al. 2013), have indicated that there should be a strong energy exchange between the dominant and subdominant components of the imbalanced KAW turbulence (Kim & Cho 2015). Even when the subdominant component was entirely absent initially, it was effectively generated in simulations during the time-scales typical for non-linear energy cascade (Cho & Kim 2016).

In this paper, we develop the first semiphenomenological model of imbalanced Alfvénic turbulence that covers the MHD, transition, and kinetic (strongly dispersive) ranges accounting for the energy exchange between the dominant and subdominant components. This paper is organized as follows: the model is formulated in Section 2; main results are obtained in Section 3; and conclusions are given in Section 4.

2 FORMULATION OF THE MODEL

Non-linear dynamic equations for AW amplitudes that cover both the MHD and dispersive ranges were derived by Voitenko (1998) in the framework of kinetic theory. If the electron inertia, as well as Landau damping, non-linear wave–particle interactions, and non-linear coupling of AWs with magnetosonic waves are ignored, these equations are (Voitenko 1998; Voitenko & De Keyser 2011)

$$\frac{\partial C_{\boldsymbol{k}}^{\sigma}}{\partial t}=\sum_{\boldsymbol{k}_1,\,\boldsymbol{k}_2}\sum_{\sigma_1,\,\sigma_2}\delta\left(\sigma\boldsymbol{k}-\sigma_1\boldsymbol{k}_1-\sigma_2\boldsymbol{k}_2\right)\exp\left(\mathrm{i}\Delta\Omega_{\boldsymbol{k}}\right)\times U_{s,s_1,s_2}(\sigma\boldsymbol{k},\sigma_1\boldsymbol{k}_1,\sigma_2\boldsymbol{k}_2)C_{\boldsymbol{k}_1}^{\sigma_1}C_{\boldsymbol{k}_2}^{\sigma_2}. \quad (1)$$

Here $C_{\boldsymbol{k}}^{\sigma}=k_\perp A_{\boldsymbol{k}}^{\sigma}$ are amplitudes of the modes, $k_\perp$ is component of the wavenumber perpendicular to the background magnetic field $\boldsymbol{B}_0$, which is assumed to be parallel to z-axis, $A_{\boldsymbol{k}}^{\sigma}$ is a component of the vector potential parallel to $\boldsymbol{B}_0$, $\Delta\Omega_{\boldsymbol{k}}=\sigma\omega_{\boldsymbol{k}}-\sigma_1\omega_{\boldsymbol{k}_1}-\sigma_2\omega_{\boldsymbol{k}_2}$, $s_i=\pm$ corresponds to the waves propagating parallel and antiparallel to the z-axis (without loss of generality in further analysis we assume that ± signs correspond to the dominant/subdominant waves), the indices $\sigma_i=\pm$ denote complex conjugate terms $C_{\boldsymbol{k}}^{+}=C_{\boldsymbol{k}}$ and $C_{\boldsymbol{k}}^{-}=C_{\boldsymbol{k}}^{*}$ (note that only real parts of the complex amplitudes, $\mathrm{Re}[C_{\boldsymbol{k}}]=(C_{\boldsymbol{k}}+C_{\boldsymbol{k}}^{*})/2$, should be considered in binary combinations), $\omega_{\boldsymbol{k}}=V_{\mathrm{A}}k_\parallel\sqrt{1+(k_\perp\rho_T)^2}$ is the frequency of KAWs, V_{A} is the Alfvén velocity, $\rho_T^2=(3/4+T_{\mathrm{e}\parallel}/T_{\mathrm{i}\perp})\rho_i^2$ at $k_\perp\rho_i<1$ and $\rho_T^2=(1+T_{\mathrm{e}\parallel}/T_{\mathrm{i}\perp})\rho_i^2$ at $k_\perp\rho_i>1$, $T_{\mathrm{e}\parallel}$ and $T_{\mathrm{i}\perp}$ are the parallel electron and perpendicular ion temperatures, and the matrix element of interaction $U_{s,s_1,s_2}(\sigma\boldsymbol{k},\sigma_1\boldsymbol{k}_1,\sigma_2\boldsymbol{k}_2)$ has the form

$$U_{s,s_1,s_2}(\sigma\boldsymbol{k},\sigma_1\boldsymbol{k}_1,\sigma_2\boldsymbol{k}_2)=\frac{V_{\mathrm{A}}k_\perp}{4\pi B_0}\Delta_{s_1,s_2}\frac{K_k}{\xi_k^2}\times\left(s_1\frac{\xi_2^2}{K_1}+s_2\frac{\xi_2^2}{K_2}+s\frac{\xi_k^2}{K_k}\right)\times\sigma_1\sigma_2\frac{(\boldsymbol{k}_1\times\boldsymbol{k}_2)_\parallel}{k_{1\perp}k_{2\perp}}. \quad (2)$$

In equation (2) we introduced notations $\xi=\rho_T k_\perp$, $K=\sqrt{1+\xi^2}$, and $\Delta_{1,2}=s_1K_1-s_2K_2$ is dimensionless phase velocity mismatch between waves 1 and 2. It has to be noted that recently the same matrix element of interaction was derived in the framework of the two-field Hamiltonian gyrofluid model for KAWs (Passot & Sulem 2019).

In the MHD limit, $K_1=K_2=1$, for co-propagating AWs ($s_1=s_2$) the phase velocity mismatch $\Delta_{1,2}=0$, i.e. there are no non-linear interactions produced by co-propagating AWs.

Further analysis of the equations describing non-linear evolution of the waves similar to equations (1) and (2) can proceed in two ways. If one is interested in weak turbulent regime (when characteristic time-scale of the non-linear interactions is much greater compared to the interacting wave periods), assuming random phases of the interactions waves and using standard technique of the weak turbulence theory one derives kinetic equation of waves that describes resonant non-linear interactions of the waves (see e.g. Voitenko 1998; Gogoberidze, Mahajan & Poedts 2009; Voitenko & De Keyser 2011, and references therein). On the other hand scaling analysis of the matrix element of interaction allows to build phenomenological models of the strong turbulence (e.g. Goldreich & Sridhar 1995; Gogoberidze 2007; Voitenko & De Keyser 2016, and references therein). Here we study the strong turbulence regime.

As it is known non-linear energy exchange among both AWs (Goldreich & Sridhar 1995) and KAWs (Voitenko 1998) is dominated by local interactions with $k_\perp\sim k_{\perp1}\sim k_{\perp2}$. Taking this into account for our phenomenological analysis we assume $|(\boldsymbol{k}_1\times\boldsymbol{k}_2)_\parallel/(k_{\perp1}k_{\perp1})|\sim 1$, $\Delta_{s_1,s_2}\sim|s_1\sqrt{1+\xi^2}-s_2|$, and

$$\frac{K_k}{\xi_k^2}\left(s_1\frac{\xi_2^2}{K_1}+s_2\frac{\xi_2^2}{K_2}+s\frac{\xi_k^2}{K_k}\right)\sim(s_1+s_2+s). \quad (3)$$

Then from equations (1) and (2) it follows that the non-linear interaction rate of co-propagating ($s=s_1=s_2$) and counter-propagating ($s_1\neq s_2$) waves is given by the following expressions (Voitenko & De Keyser 2016):

$$\gamma_{k\pm}^{\mathrm{NL}}(p)\sim\frac{2+p}{4\pi}k_\perp V_{\mathrm{A}}\Delta_{1,p}\frac{B_{(\pm p)k_\perp}}{B_0}, \quad (4)$$

where $p=1$ corresponds to co-collisions ($s=s_1=s_2$) and $p=-1$ corresponds to the collisions of counter-propagating waves. In equation (4), $B_{\pm k_\perp}$ denotes typical amplitudes of the dominant and subdominant perturbations with the perpendicular length scale $1/k_\perp$.

Similarly, non-linear interaction of co-propagating waves generates the wave propagating in the opposite direction. Under simplifying assumptions made above for the non-linear exchange rates between counter-propagating waves, we obtain

$$\gamma_{k\pm}^{\mathrm{EX}}\sim\frac{1}{4\pi}k_\perp V_{\mathrm{A}}\Delta_{1,1}\frac{B_{\mp k_\perp}^2}{B_{\pm k_\perp}B_0}=\gamma_{k\pm}^{\mathrm{NL}}(1)\frac{B_{\mp k_\perp}}{3B_{\pm k_\perp}}. \quad (5)$$

As will be shown below the latter process causes significant enhancement of the subdominant wave energy and is responsible for the generation of subdominant waves, even if they are absent initially. Before proceeding further we shortly consider MHD limit and the model developed by Voitenko & De Keyser (2016).

2.1 The MHD limit

In the MHD limit $\xi=\xi_1=\xi_2=0$, the phase velocity mismatches between co-propagating waves $\Delta_{1,\,1}=0$, therefore $\gamma_{k\pm}^{\mathrm{NL}}(1)=\gamma_{k\pm}^{\mathrm{EX}}=0$ and there are no non-linear interactions produced by co-propagating AWs. In the case under consideration there is no energy exchange between the dominant and subdominant components (Goldreich & Sridhar 1995). The non-linear interaction rates between counter-propagating AWs (given by equation 4 with $p=-1$) now take the form

$$\gamma^{\mathrm{NL}}_{k\pm}(-1) \sim V_{\mathrm{A}} \frac{k_\perp B_{\mp k_\perp}}{B_0}. \tag{6}$$

In general case non-linear interaction rates $\gamma^{\mathrm{NL}}_{k\pm}(-1)$ do not equal to the corresponding cascade rates $\gamma^{\mathrm{CAS}}_{k\pm}$ of counter-propagating waves and some assumption regarding parallel correlation lengths of the interacting AW packets is necessary to determine the cascade rates and correspondingly the energy spectra of the counter-propagating AWs (Lithwick et al. 2007; Beresnyak & Lazarian 2008; Candran 2008). In general case the cascade rate is determined as (Matthaeus, Oughton & Zhou 2009)

$$\gamma^{\mathrm{CAS}}_{k\pm}(-1) \sim \frac{\left[\gamma^{\mathrm{NL}}_{k\pm}(-1)\right]^2}{k^{\mp}_{\parallel} V_{\mathrm{A}}}, \tag{7}$$

where $1/k^{\mp}_{\parallel}$ are parallel correlation lengths (at corresponding perpendicular length scales $1/k_\perp$) of subdominant and dominant waves, respectively. In this paper, we consider the model developed by Lithwick et al. (2007). According to this model the cascades of the both dominant and subdominant components are strong, i.e. non-linear interaction time-scales are comparable to the linear propagation time-scales $\gamma^{\mathrm{NL}}_{k\pm}(-1) \sim k^{\mp}_{\parallel} V_{\mathrm{A}}$, so in this case

$$\gamma^{\mathrm{CAS}}_{k\pm}(-1) \sim \gamma^{\mathrm{NL}}_{k\pm}(-1), \tag{8}$$

as it should be the case for strong turbulence regime. Taking into account relations for energy spectra $E_\pm(k_\perp)k_\perp \sim B^2_{\pm k_\perp}/(4\pi)$, relations for energy cascade rates

$$\varepsilon_\pm = \gamma^{\mathrm{CAS}}_{k\pm}(-1) E_\pm(k_\perp), \tag{9}$$

and using equation (6), we conclude that in the considered case the both components follow Kolmogorov scaling $E_\pm(k_\perp) \sim k_\perp^{-5/3}$ with the following relation between the energy cascade rates and amplitudes:

$$\frac{\epsilon_+}{\epsilon_-} = \frac{B_{+k_\perp}}{B_{-k_\perp}}. \tag{10}$$

Note that in the strongly imbalanced case $\epsilon = \epsilon_+ + \epsilon_- \approx \epsilon_+$ and $E_{\mathrm{tot}}(k_\perp) = E_+(k_\perp) + E_-(k_\perp) \approx E_+(k_\perp)$, equations (9) and (10) yield

$$\epsilon \sim k_\perp E_{\mathrm{tot}}(k_\perp)^{3/2} \frac{\epsilon_-}{\epsilon_+}. \tag{11}$$

This relation means that with increasing imbalance (i.e. decreasing ϵ_-/ϵ_+) the turbulent cascade weakened, in the sense that to keep the same cascade rate ϵ at some wavenumber $k_\perp$ the energy density (and therefore the amplitude $B_{+k_\perp}$) should increase.

2.2 MHD–kinetic transition with the constant energy fluxes

When $k_\perp$ increases AWs become dispersive, $\Delta_{1,1} \neq 0$ and non-linear interactions between co-propagating KAWs also contribute to the energy transfer to the smaller scales ($\gamma^{\mathrm{NL}}_{k\pm}(1) \neq 0$). It can be shown that in the considered model non-linear interactions between co-propagating waves also follow strong turbulence phenomenology $[\gamma^{\mathrm{NL}}_{k\pm}(1) \sim \gamma^{\mathrm{CAS}}_{k\pm}(1)]$ and the equations describing relations between the energy fluxes and energy densities are (Voitenko & De Keyser 2016)

$$\epsilon_- = \left[\gamma^{\mathrm{NL}}_{k-}(1) + \gamma^{\mathrm{NL}}_{k-}(-1)\right] \frac{B^2_{-k_\perp}}{4\pi}, \tag{12}$$

$$\epsilon_+ = \left[\gamma^{\mathrm{NL}}_{k+}(1) + \gamma^{\mathrm{NL}}_{k+}(-1)\right] \frac{B^2_{+k_\perp}}{4\pi}. \tag{13}$$

If the turbulence is strongly imbalanced, $\epsilon_+/\epsilon_- \gg 1$, then from equation (4) follows that $\gamma^{\mathrm{NL}}_{k+}(1)$ increases with $k_\perp$ faster than $\gamma^{\mathrm{NL}}_{k+}(-1)$ and the turbulence cascade eventually arrives to the wavenumber $k_{\perp *}$,

$$k_{\perp *}\rho_T \approx \sqrt{\frac{\epsilon_-}{\epsilon_+}} \ll 1, \tag{14}$$

where $\gamma^{\mathrm{NL}}_{k+}(1)$ becomes equal to $\gamma^{\mathrm{NL}}_{k+}(-1)$. At larger wavenumbers, in the range $\sqrt{\epsilon_-/\epsilon_+} < k_\perp \rho_T < 1$, a specific weakly dispersive regime (WDR) of the Alfvénic turbulence can be formed by non-linear interactions among co-propagating dominant waves. From equation (13) it follows that in this regime the spectrum should be very steep, $E_+(k_\perp) \sim k_\perp^{-3}$ (Voitenko & De Keyser 2016). In the strongly dispersive regime (SDR), at $k_\perp \rho_T \geq 1$, the scaling analysis of equations (12) and (13) predicts the standard spectra $E_\pm(k_\perp) \sim k_\perp^{-7/3}$ of the strong KAW turbulence.

Another consequence of the equations (12) and (13) is that if subdominant component and therefore corresponding energy flux $\epsilon_- = 0$ is absent at some scale $k_{\perp 1}$, then corresponding energy spectrum $E_-(k_\perp) = 0$ remains zero for all $k_\perp > k_{\perp 1}$.

2.3 Model with the non-constant cascade rates

The model described by equations (12) and (13) assumes that the energy fluxes of dominant and subdominant waves are constant not only in the MHD but also in dispersive range. However, the energy fluxes ϵ_+ and ϵ_- are constant separately only in the MHD limit (Goldreich & Sridhar 1995). In the case of dispersive KAWs, there are several mechanisms of the energy exchange between the counter-propagating wave fractions both in the weak and strong turbulence regimes (Voitenko 1998; Voitenko & De Keyser 2011; Passot & Sulem 2019, and references therein).

In the framework of the approach developed in this paper equation (5) shows that the dominant co-propagating KAWs generate subdominant waves, and vice versa. Moreover, this kind of energy exchange has been observed in electron MHD numerical simulations of the KAW turbulence in the SDR (Cho & Kim 2016): even when the subdominant component was *initially absent*, it was effectively generated by the dominant component during the typical cascade time-scale. Solar wind observations also suggest such energy exchange (see e.g. fig. 1b by Wicks et al. 2011 showing the turbulence imbalance decreasing at small dispersive scales).

The main effect of these exchange processes is that the energy fluxes ϵ_+ and ϵ_- through different scales are not constant any more. The non-linear interactions among dominant waves transfer the energy to smaller scale co-propagating waves, but a fraction of their energy is transferred to the counter-propagating subdominant waves. To incorporate the energy exchange effects in the phenomenological model, we add the corresponding terms to equations (12) and (13):

$$\epsilon_- + \gamma^{\mathrm{EX}}_{k-} \frac{B^2_{-k_\perp}}{4\pi} - \gamma^{\mathrm{EX}}_{k+} \frac{B^2_{+k_\perp}}{4\pi} = \left[\gamma^{\mathrm{NL}}_{k-}(1) + \gamma^{\mathrm{NL}}_{k-}(-1)\right] \frac{B^2_{-k_\perp}}{4\pi}, \tag{15}$$

$$\epsilon_+ - \gamma^{\mathrm{EX}}_{-k_\perp} \frac{B^2_{-k_\perp}}{4\pi} + \gamma^{\mathrm{EX}}_{k+} \frac{B^2_{+k_\perp}}{4\pi} = \left[\gamma^{\mathrm{NL}}_{k+}(1) + \gamma^{\mathrm{NL}}_{k+}(-1)\right] \frac{B^2_{+k_\perp}}{4\pi}. \tag{16}$$

The left-hand sides of these equations represent the scale-dependent fluxes of counter-propagating waves. Because of the energy exchange they are not constant separately, but, as is expected in the dissipationless case, their sum remains constant.

In the next section, we use equations (15) and (16) to study how the energy exchange between dominant and subdominant components affects the Alfvénic turbulence in WDR and SDR.

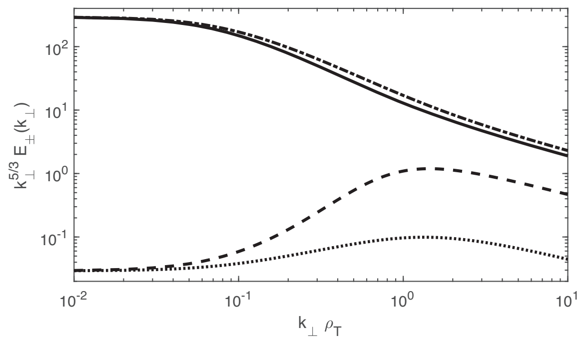

Figure 1. Kolmogorov-compensated energy spectra $E_{\pm}(k_{\perp})k_{\perp}^{5/3}$ with and without energy exchange terms. The solid and the dashed lines correspond, respectively, to the compensated spectra of the dominant and subdominant components obtained by the solution of equations (15) and (16), and the dash–dotted and dotted lines correspond to the same energy spectra obtained by the solution of equations (12) and (13).

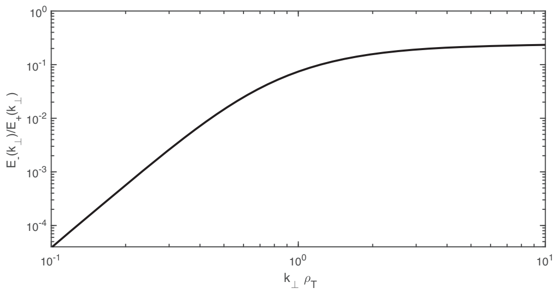

Figure 2. The ratio of subdominant and dominant energies for $\epsilon_- = 0$ as a function of the normalized wavenumber $k_{\perp}\rho_T$.

3 RESULTS AND DISCUSSION

In this section, we present results of the numerical solution of the algebraic equations (12) and (13) and (15) and (16) with respect to $B_{\pm k_\perp}$ for given values of $\varepsilon_\pm$ at various $k_\perp$. Fig. 1 shows the Kolmogorov-compensated energy spectra $E_{\pm}(k_{\perp})k_{\perp}^{5/3} = B^2_{\pm k_\perp}k_{\perp}^{2/3}/(4\pi)$ in the strongly imbalanced case with $\epsilon_+/\epsilon_- = 100$. The solid and the dashed lines represent the compensated energy spectra of the components obtained by solution of equations (15) and (16), and the dashed–dotted and dotted lines correspond to the compensated energy spectra obtained by solution of equations (12) and (13).

Obviously, as can be seen in Fig. 1, in the presence of the energy exchange the subdominant waves are generated much more efficiently, but a significant imbalance still exists. There is also one more non-trivial feature that can be seen in Fig. 1. In the strongly dispersive limit $k_\perp\rho_T \gg 1$, the scalings of the both dominant and subdominant wave amplitudes are the same and lead to the same spectral index $-7/3$. This result is clearly non-trivial because the energy fluxes within two counter-propagating wave components are not constant and therefore no self-similar behaviour can be expected a priori. In our case this feature is related to the fact that all terms in equations (15) and (16) have the same scaling. This possibility is not unique, the scaling law of the turbulence with non-conservative fluxes can result from the balance between the non-linear cascade and an anisotropic dissipation (Gogoberidze 2005).

Fig. 2 shows the ratio of subdominant and dominant energies $E_-(k_\perp)/E_+(k_\perp)$ for $\epsilon_- = 0$ as a function of the normalized wavenumber $k_\perp\rho_T$. As can be seen from the Fig. 2, even in the absence of the subdominant component for $k_\perp\rho_T \ll 1$, the energy of the subdominant waves reach the limit of $\approx$25 per cent of the total energy. This result qualitatively agrees with the results of recent numerical simulations of electron MHD (Cho & Kim 2016).

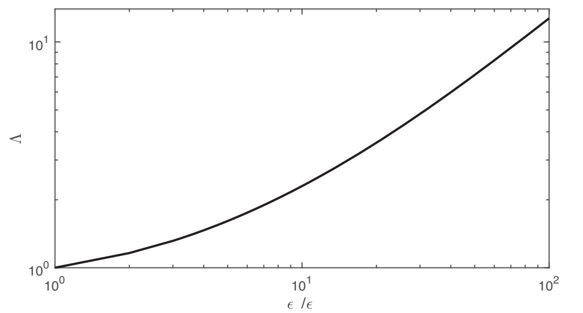

Figure 3. The ratio Λ of the subdominant component energies $E_-(k_\perp)$ with and without energy exchange between the dominant and subdominant components.

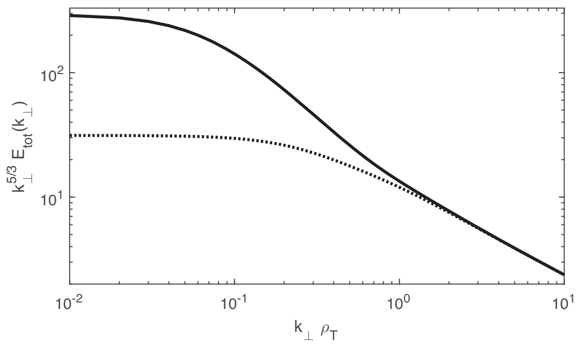

Figure 4. Compensated energy spectrum of the waves with and without energy exchange as a function of normalized wavenumber $k_\perp\rho_T$. Solid line corresponds to the case with energy exchange – solution of equations (12) and (13) with $\epsilon_+/\epsilon_- = 100$. Dash–dotted line represents the solution of equations (15) and (16), i.e. ignoring energy exchange between different wave modes. The dotted line corresponds to the symmetric case $\epsilon_+ = \epsilon_-$.

Fig. 3 shows the ratio Λ of the subdominant component energies $E_-(k_\perp)$ obtained by solution of the equations (15) and (16) and (12) and (13) for $k_\perp\rho_T \gg 1$. From this figure it can be seen that the energy exchange causes significant enhancement of the subdominant component energy in SDR especially for strongly imbalanced turbulence, $\epsilon_+/\epsilon_- \gg 1$.

Fig. 4 shows the total energy spectrum $E_{\rm tot}(k_\perp)$ of the turbulence as a function of the normalized wavenumber $k_\perp\rho_T$ obtained by the solution of equations (15) and (16) with the fixed energy cascade rate $\epsilon = \epsilon_+ + \epsilon_-$ but with different levels of imbalance. The solid line corresponds to the strongly imbalanced case ($\epsilon_+/\epsilon_- = 100$), whereas dotted line corresponds to the case $\epsilon_+/\epsilon_- = 4$. As it follows from equation (14), the width of the transition interval between the MHD and strongly dispersive KAW regimes has the order $k_\perp\rho_T \sim \sqrt{\epsilon_+/\epsilon_-}$. Indeed, as can be seen in Fig. 4, the width of the transition interval is much wider when the imbalance is stronger. It is well known (Lithwick et al. 2007; Beresnyak & Lazarian 2008; Perez & Boldyrev 2009) and follows from equation (11) that in MHD turbulence increase of the imbalance reduces the energy cascade rate because the cascade rate of the dominant component is proportional to the shears produced by the weaker subdominant waves. Then, to keep the same cascade rate, the energy density should be higher, which is indeed demonstrated in Fig. 4 for small values of the normalized wavenumber $k_\perp\rho_T$. However, this is not the

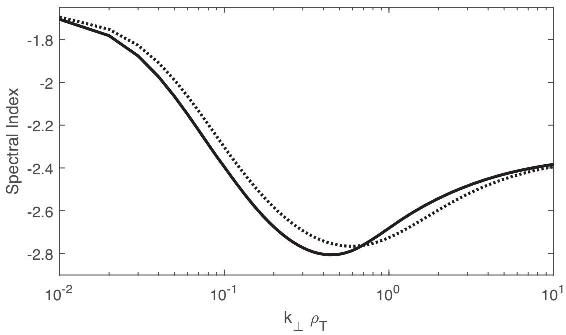

Figure 5. Local value of the spectral index of the dominant component of the imbalanced Alfvénic turbulence with (solid line) and without (dotted line) energy exchange between counter-propagating wave modes for $\epsilon_+/\epsilon_- = 100$.

case for strongly dispersive KAW turbulence at $k_\perp\rho_T > 1$, because in these wavenumbers the energy cascade is generated by the non-linear interactions involving both the co- and counter-propagating waves.

Fig. 5 shows the local spectral index of the dominant component of the imbalanced Alfvénic turbulence with (solid line) and without (dotted line) energy exchange between the dominant and subdominant components for $\epsilon_+/\epsilon_- = 100$. Note that the model developed by Voitenko & De Keyser (2016) predicts that for $\epsilon_+/\epsilon_- \to \infty$ an asymptotic self-similar regime should develop where the turbulence cascade is dominated by the non-linear collisions among co-propagating dominant waves. As a result, the dominant spectrum in WDR becomes steeper with the spectral index approaching -3. This behaviour is qualitatively reproduced in Fig. 5 showing that the spectrum becomes significantly steeper in WDR. However, the predicted index -3 can be realized only for very large imbalances $\epsilon_+/\epsilon_- \geq 10^4$.

4 CONCLUSIONS

We developed the semiphenomenological model for the imbalanced Alfvénic turbulence taking into account the energy exchange between counter-propagating waves that makes the energy fluxes scale-dependent in the dispersive range. The model covers both the MHD and two dispersive ranges, WDR and SDR.

In the dispersive transition range a significant fraction of the dominant wave energy is transferred to the oppositely propagating subdominant component in accordance with the recent numerical results (Cho & Kim 2016). In spite of the intense energy transfer from dominant to subdominant component, total equipartition of the energy densities is not reached. The energy exchange between dominant and subdominant components significantly changes the local spectral index in the WDR at the scales well before the ion gyroscale is reached. Our results are also consistent with the behaviour of the power spectra and turbulence imbalance in the solar wind turbulence observed by Wicks et al. (2011).

In the strongly dispersive wavenumber range, $k_\perp\rho_T \gg 1$, both the dominant and subdominant components of the turbulence follow the same power law $E_\pm(k_\perp) \sim k_\perp^{-7/3}$ regardless of the fact that the dominant and subdominant energy fluxes are not conserved separately.

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DATA AVAILABILITY

The authors confirm that the data supporting the findings of this study are available within this paper.

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