THE OBLATENESS EFFECT ON THE SOLAR RADIATION INCIDENT AT THE TOP OF THE ATMOSPHERE OF MARS

E. Van Hemelrijck

Belgian Institute for Space Aeronomy
Brussels, Belgium

ABSTRACT

The daily solar radiation incident at the top of the atmosphere of Mars, with and without the effect of the oblateness, is calculated as a function of season and latitude. It is shown that for parts of the summer, the daily insolation of Mars, assumed as an oblate planet, is slightly increased. In winter, the flattening effect results in a somewhat more extensive polar region, the solar energy input being always reduced (by more than 5% near the poles). It is found that the mean summer daily insolation is scarcely increased between the equator and the subsolar point, but decreased poleward of the above mentioned limit. In winter, however, the mean daily insolation is always reduced, the maximum loss of insolation attaining approximately 2% in the 60-80° latitude interval.

Keywords : Mars, solar radiation, oblateness effect.

1. INTRODUCTION

In studies involving solar radiation problems, the instantaneous insolation, defined as the solar heat flux sensed at a given time by a horizontal unit area of the upper boundary of the atmosphere at a given point on the planet and per unit time, constitutes a very important input data.This amount of solar energy and its variability with latitude and time is mainly governed by two factors : on one hand, the solar flux as a function of the orbital distance and, on the other hand, the cosine of the zenith angle of the incident solar radiation.

Considering more particularly the latter parameter, it should be pointed out that for a planet assumed to be spherical, the zenith distance may be expressed as a simple function of latitude, solar declination and local hour angle of the Sun, the radius vector coinciding with the normal to the horizon plane. For an oblate, however, there is an angle (vanishing at the equator and the poles) between the two directions mentioned above. This so-called angle of the vertical is dependent upon the flattening.

Although the oblateness effect on the solar radiation incident at the top of the outer planets Jupiter, Saturn, Uranus and Neptune (Ref. 1) and of the Earth (Ref. 2) has been treated recently in detail, it should be emphasized that the effect of the flattening on the upper-boundary insolation of Mars, the outer planet having the smallest flattening, has never been studied. This paper, therefore, analyzes this effect.

Our results are presented in the form of a contour map, illustrating the latitudinal and seasonal variation of the ratio of the daily insolation with and without the influence of the oblateness, and in a diagram giving the latitudinal variation of the numerical difference of both insolations corresponding to the southern and northern winter hemisphere respectively. For the sake of completeness we have included a figure showing the percentage difference of the mean (summer, winter and annual) daily insolations as a function of latitude.

2. DAILY INSOLATION WITH AND WITHOUT THE OBLATENESS EFFECT

In this section we only briefly will mention the major expressions needed for the computation of the daily insolation with and without the oblateness effect. For more details see e.g. Refs. 1-8.

The instantaneous insolation I at the upper-boundary of a planet can be expressed as

$$I = S \cos z \qquad (1)$$

with $$S = S_o/r_s^2 \qquad (2)$$

and $$r_s = a_s(1-e^2)/(1+e \cos W) \qquad (3)$$

where z is the zenith angle of the incident solar radiation, S is the solar flux at an heliocentric distance r_s and S_o is the solar constant at the mean Sun-Earth distance of 1AU taken at 1353 Wm^{-2} or 1.94 cal cm^{-2}(min)$^{-1}$ (Ref. 9). Furthermore, in Eq. 3, a_s, e and W are respectively the planet's semi-major axis, the eccentricity and the true anomaly which is given by

$$W = \lambda_s - \lambda_p \tag{4}$$

where λ_s and λ_p are the planetocentric longitude of the Sun and the planetocentric longitude of the planet's perihelion. The numerical values of the parameters used for the calculations are listed in Table 1. In this table one can find also the obliquity ε, the equatorial radius a_e, the polar radius a_p, the flattening $f = (a_e-a_p)/a_e$ and the sidereal period of axial rotation T (sidereal day). Note that the elements of the planetary orbit and the dimensions of Mars are taken from Refs. 5 and 10.

$$\cos Z = \cos v \cos z + \sin v(- \tan \phi' \cos z + \sin \delta_s \sec \phi') \tag{10}$$

The daily insolation of an oblate planet I_{DO} can now be found by integrating Eq. 1 within the appropriate time limits, where cos z has to be replaced by Eq. 10 yielding

Table 1.
Elements of the planetary orbit and dimensions of Mars

a_s (AU)	e	λ_p (°)	(°)	a_e (km)	a_p (km)	f	T (Earth days)
1.524	0.09339	248	25.20	3397.0	3379.5	0.00515	1.02

For a spherical planet, z may be expressed as

$$\cos z = \sin \phi' \sin \delta_s + \cos \phi' \cos \delta_s \cos h \tag{5}$$

where ϕ' is the geocentric latitude (which equals the geographic latitude ϕ), δ_s is the solar declination and h is the local hour angle of the Sun. Furthermore, the solar declination can be calculated using the following expression

$$\sin \delta_s = \sin \varepsilon \sin \lambda_s \tag{6}$$

The daily insolation I_D can now be obtained by integrating Eq. 1 over daytime assumed to be equal to the time that elapses between rising and setting of the Sun and is given by

$$I_D = (ST/\pi)(h_o \sin \phi' \sin \delta_s + \sin h_o \cos \phi' \cos \delta_s) \tag{7}$$

where h_o is the sunset (or sunrise) hour angle and may be determined from Eq. 5 by the condition that at sunset (or sunrise) cos z = 0. Hence

$$h_o = \arccos(- \tan \delta_s \tan \phi') \tag{8}$$

if

$$|\phi'| < \pi/2 - |\delta_s|$$

In regions where the Sun does not rise ($\phi' < - \pi/2 + \delta_s$ or $\phi' > \pi/2 + \delta_s$) we have $h_o = 0$; in regions where the Sun remains above the horizon all day ($\phi' > \pi/2 - \delta_s$ or $\phi' < - \pi/2 - \delta_s$) we may put $h_o = \pi$.

In the case of an oblate planet there is an angle $v = \phi-\phi'$, the so-called angle of the vertical, between the radius vector and the normal to the horizon plane; it vanishes at the equator and the poles while elsewhere $\phi > \phi'$ numerically. The angle v is dependent upon the geocentric latitutude ϕ' and the flattening f by the relationship

$$v = \arctan\left[(1-f)^2 \tan \phi'\right] - \phi' \tag{9}$$

Denoting the zenith distance for an oblate planet by Z, the following relation can easily be obtained by applying the formulae of spherical trigonometry

$$I_{DO} = (ST/\pi)\Big\{\cos v(h_{oo} \sin \phi' \sin \delta_s + \sin h_{oo} \cos \phi' \cos \delta_s) + \sin v\big[- \tan \phi' (h_{oo} \sin \phi' \sin \delta_s + \sin h_{oo} \cos \phi' \cos \delta_s) + h_{oo} \sin \delta_s \sec \phi'\big]\Big\} \tag{11}$$

where h_{oo}, the local hour angle at sunset (or sunrise) for an oblate planet, is generally slightly different from h_o. As for a spherical planet, h_{oo} may be derived from Eq. 10 by putting cos Z = 0. After some rearrangements, and as expected, the expression for h_{oo} in terms of δ_s and ϕ is found to be similar to Eq. 8.

Taking into account Eqs. 7 and 11, the ratio I_{DO}/I_D at the top of Mars can now easily be determined as a function of geocentric latitude ϕ' and solar longitude δ_s.

The mean (summer, winter and annual) daily insolations, hereafter denoted as $(\bar{I}_D)_S$, $(\bar{I}_D)_W$ and $(\bar{I}_D)_A$ (spherical planet) and $(\bar{I}_{DO})_S$, $(\bar{I}_{DO})_W$ and $(\bar{I}_{DO})_A$ (oblate planet) respectively, may be found by integrating numerically Eqs. 7 and 11 within the appropriate time limits, yielding the total insolation over a season or a year and by dividing the obtained result by the corresponding length of the season T_S = 381.3 (summer) and T_W = 305.6 (winter) Earth days or by the sidereal period of revolution or tropical year (T_o = 686.9 Earth days).

Note that $(\bar{I}_D)_A$ and $(\bar{I}_{DO})_A$ can also directly be computed from the knowledge of $(\bar{I}_D)_S$, $(\bar{I}_D)_W$ and $(\bar{I}_{DO})_S$, $(\bar{I}_{DO})_W$. Indeed, taking into account the numerical values of T_S/T_o and T_W/T_o, the average yearly insolations can, in a very good approximation, be written under the following general form

$$(\bar{I}_D)_A = 0.56\ (\bar{I}_D)_S + 0.44\ (\bar{I}_D)_W \tag{12}$$

and

$$(\bar{I}_{DO})_A = 0.56(\bar{I}_{DO})_S + 0.44(\bar{I}_{DO})_W \tag{13}$$

3. DISCUSSION OF THE RATIO DISTRIBUTION OF THE DAILY INSOLATIONS

In two previous works, dealing with the oblateness effect on the solar radiation incident at the top of the atmospheres of the outer planets Jupiter, Saturn, Uranus and Neptune (Ref. 1) and of the Earth (Ref. 2), we studied qualitatively some characteristic features of the ratio distribution I_{DO}/I_D in the northern hemisphere both for summer ($0° < \lambda_s < 180°$) and winter ($180° < \lambda_s < 360°$) period. It should, however, be emphasized that the results presented are also valid for the southern hemisphere and that they are evidently applicable to Mars. The following are the major conclusions that were reached :

(1) In summer and in the region of permanent sunlight ($h_{oo} = h_o = \pi$) the isocontours I_{DO}/I_D parallel the lines of constant geocentric latitude ϕ' and the daily solar radiation I_{DO} is greater than I_D. The maximum value of the ratio I_{DO}/I_D can be expressed by the following relationship

$$(I_{DO}/I_D)_{max} = \sin\left\{\text{arc tan}\left[(1-f)^2 \times \text{cotan}\,\varepsilon\right]\right\}/\cos\varepsilon \qquad (14)$$

(2) In summer and in the region limited by the seasonal march of the Sun, the solar radiation of an oblate planet (I_{DO}) is increased with respect to the insolation of a spherical one (I_D).

(3) For latitudes between the subsolar point and the region where the Sun remains above the horizon all day, the ratio cos Z/cos z is decreasing (with cos Z < cos z), whereas h_{oo}/h_o is increasing (with $h_{oo} > h_o$). Whether or not the regions mentioned in (1) and (2) are linked depends on the relative effect of those two ratios, both being function of f and ε, and can only be evaluated by computation of the expression I_{DO}/I_D.

(4) In winter, the effect of the flattening results in a more extensive polar region, the insolation is always reduced ($I_{DO} < I_D$) and the curves of constant ratio I_{DO}/I_D roughly parallel the boundary of the polar night except in the neighborhood of the equinoxes.

Application of Eqs. 7 and 11 leads to the isocontour map illustrated in Figure 1, where values of constant ratio distribution I_{DO}/I_D are given on each curve. Solar declination (lower part) and the region where the Sun does not set (upper part) are indicated by the dashed lines. The area of permanent darkness is shaded and the region of enhanced solar radiation ($I_{DO} > I_D$) is dotted.

From Figure 1 it can be seen that in the region of permanent sunlight the incoming solar radiation (I_{DO}) is increased when compared to I_D. Furthermore, it follows from Eq. 14 that in the region considered $(I_{DO}/I_D)_{max} = 1.0019$ ($\simeq 0.2\%$). This extremely small gain of insolation is particularly due to the even small value of the flattening. For comparaison, it is instructive to note that $(I_{DO}/I_D)_{max}$ is equal to 1.0010, 1.0003, 1.037, 1.124 and 1.010 for the Earth, Jupiter, Saturn, Uranus and Neptune respectively. The effect of parallelism between the isocontours I_{DO}/I_D and the lines of constant geocentric latitude ϕ' is not illustrated in

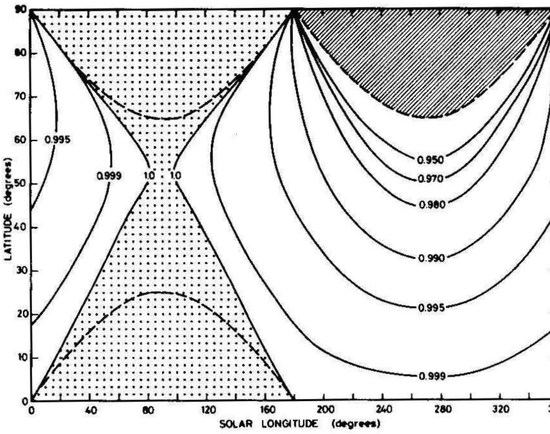

Figure 1. Seasonal and latitudinal variation of the ratio I_{DO}/I_D of the daily insolation with (I_{DO}) and without the oblateness effect at the top of the atmosphere of Mars. Solar declination (lower part) and the region where the Sun does not set (upper part) are indicated by the dashed lines. The area of permanent darkness is shaded, whereas the region of enhanced solar radiation ($I_{DO} > I_D$) is dotted. Values of constant ratio distribution I_{DO}/I_D are given on each curve.

Figure 1. The reason for this non-representation lies in the fact that the numerical value of I_{DO}/I_D is negligible small (1.0012 and 1.0003 at ϕ' = 70 and 80° respectively.

It can mathematically be proved that, in summer, in the region bounded by the equator and the solar declination curve, both the length of the day and the cosine of the zenith distance are enhanced by the effect of the flattening. Hence, it follows that in this particular region, the solar energy input at the top of the atmosphere of Mars, assumed as an oblate, is increased with respect to the insolation of a spherical planet.

Figure 1 also reveals that the two zones where $I_{DO} > I_D$ are linked by two curves coinciding remarkably well with the two branches of an hyperbola. Outside the region of increased solar radiation the loss of insolation depends upon the solar longitude. For example, if, at ϕ' = 45°, λ_s increases from 0 to approximately 50°, the ratio I_{DO}/I_D increases from 0.995 ($\simeq 0.5\%$) to about 0.999 ($\simeq 0.1\%$). For the sake of clearness it has to be pointed out that, in summer, the increase or decrease in solar energy is significantly small.

As for all planets (Ref. 1-2) the polar region is extended. It should, however, be emphasized that the Artic circles $I_{DO} = 0$ and $I_D = 0$ practically coincide, the maximum difference attaining scarcely 0°.23 at a solar longitude of 270°. This value is higher than the one for the Earth (0°.14) but is considerably lower than those for the other planets varying from about 0.4 (Jupiter) to approximately 5° (Saturn). It is clear that this finding is ascribed to the extremely small flattening.

Figure 1 shows that in winter, as stated earlier, the insolation I_{DO} is always reduced when compared to I_D. The incoming solar energy slowly decreases in passing from equator latitudes to midlatitudes; it is found that at winter solstice and if ϕ' increases from 10 to about 55° the ratio I_{DO}/I_D decreases from 0.999 ($\simeq$ 0.1%) to approximately 0.950 ($\simeq$ 5%). At higher latitudes, particularly near the region where the Sun does not rise, I_{DO}/I_D drops very rapidly to zero. Although the loss of insolation is not very striking it follows from Figure 1 that for parts of the winter the incident solar radiation is decreased by more than 5% through the flattening.

Another point about the curves is that they roughly parallel the limit of the polar night exept, of course, in the vicinity of the equinoxes. Finally, it is also suitable to remark that in summer, respectively in winter, the curves of constant ratio I_{DO}/I_D are perfectly symmetric with respect to the summer and winter solstices.

4. DISCUSSION OF THE NUMERICAL DIFFERENCE DISTRIBUTION OF THE DAILY INSOLATIONS

In Figure 2 we have plotted the numerical difference [in cal cm^{-2} (day)$^{-1}$] of the daily solar radiation, without (I_D) and with (I_{DO}) the oblateness effect, as a function of latitude corresponding to the southern and northern winter hemisphere respectively.

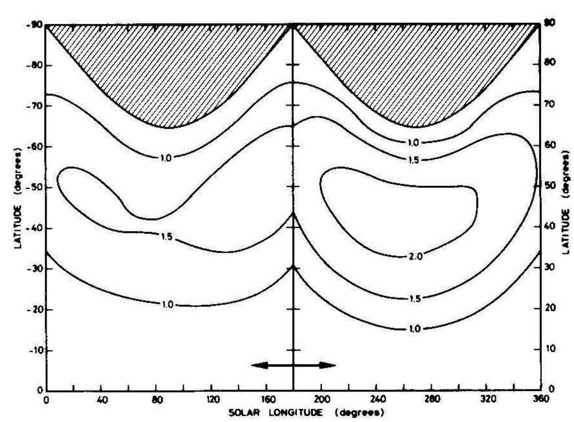

Figure 2. Latitudinal variation of the numerical difference (I_D - I_{DO}) of the daily insolations corresponding to the southern (left) and northern (right) winter hemisphere. The areas of permanent darkness are shaded. Values of constant numerical difference distribution, in cal cm^{-2} (day)$^{-1}$, are given on each curve.

Although the isocontours representing the ratio distribution of both insolations (I_{DO}/I_D) are perfectly identical in both hemispheres, it follows from the north-south seasonal asymmetry in the daily solar insolation produced by the eccentricity of the orbit that the absolute loss of solar radiation as a function of latitude and solar longitude is different in both hemispheres. Figure 2 clearly indicates that the decrease of solar energy in the northern winter hemisphere is obviously higher than in the southern one reaching a maximum value as much as 2 cal cm^{-2} (day)$^{-1}$ at midlatitudes; in the southern hemisphere the maximum loss of insolation is about 1.5 cal cm^{-2} (day)$^{-1}$.

5. DISCUSSION OF THE MAIN DAILY INSOLATIONS

In Figure 3 the influence of the flattening is plotted in terms of the percentage difference $100(\bar{I}_{DO} - \bar{I}_D)/\bar{I}_D$ as a function of geocentric latitude. As in section 2, the bars over symbols signify seasonal or annual averages.

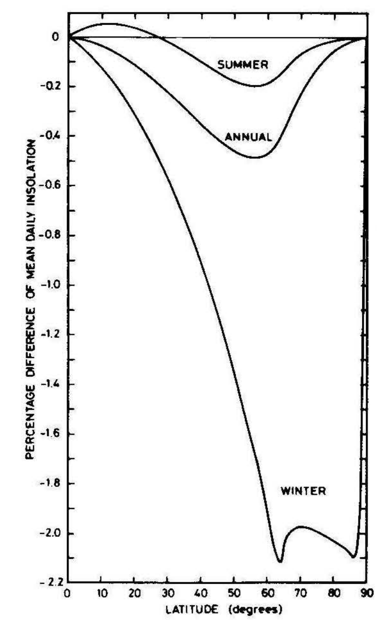

Figure 3. Latitudinal variation of the percentage difference $[100(\bar{I}_{DO} - \bar{I}_D)/\bar{I}_D)]$ of the mean (summer, winter and annual) daily insolations. The bars over symbols signify seasonal and annual averages.

Concerning more particularly the mean summertime insolation, the influence of the flattening, although very small, is obviously evident from Figure 3. Indeed, in the latitude interval (50-60°) it can be seen that about 0.2% of the mean summertime insolation is lost through the oblateness effect. Outside this interval, the effect is of decreasing significance. Another interesting phenomenon is that for latitudes between the equator and the subsolar point, the main daily summer insolation of Mars, assumed as an oblate planet, is increased (Refs. 1-2 and 6). However, owing to the small flattening, the rise of insolation is practically negligible, reaching only a maximum value of about 0.05% at a latitude of approximately 10°.

During the winter season, as stated previously, the daily insolation I_{DO} is always reduced with respect to I_D; consequently $(\bar{I}_{DO})_W < (\bar{I}_D)_W$ at any latitude. Figure 3 illustrates that in winter the loss of insolation is of most importance between 45 and 85° where a percentage difference as much as 1% has been found with a maximum value near 2% in the 60-80° latitude interval.

The partial gain of the mean summertime insolation equatorward of the subsolar point being considerably lower than the corresponding loss of insolation in winter evidently results in a mean annual daily insolation which is reduced over the entire latitude interval as shown in Figure 3. It can be seen that the daily insolation averaged over a one year cycle is decreased by about 0.5% at mid-latitudes.

6. CONCLUDING REMARKS

In the present paper, we have investigated the influence of the flattening on the solar energy input at the top of the atmosphere of Mars. As a result of this study we can drawn the general conclusion that the effect of the oblateness causes non-negligible, although relatively small, variations in both the planetary-wide distribution and the intensity of the daily solar radiation.

For parts of the summer, the daily insolation of Mars, assumed as an oblate planet, is slightly increased. The maximum increase of the incoming solar radiation, occuring at a geocentric latitude of about 65° near summer solstice is approximately equal to 0.2%. Outside the dotted zone of Figure 1, the direct solar radiation I_{DO} is decreased when compared to I_D. For example, in the neighborhood of the equinoxes the loss of insolation ranges from 0.1 to 0.5% (except at equator latitudes), whereas elsewhere the oblateness effect is either unimportant.

In winter, the influence of the flattening causes the polar region to enlarge over an extremely small distance of about 15 km at winter solstice. The insolation is always reduced, the rate of decrease depending to a large extent on the geocentric latitude. For comparison, at $\lambda_s = 270°$, the loss of solar energy amounts to about 0.1, 0.5, 1.0, 2.0, 3.0 and 5.0% respectively at latitudes of approximately 5, 20, 30, 45, 50 and 55°. Moreover, in the relatively small area limited by the iso-contour $I_{DO}/I_D = 0.950$ and the region of permanent darkness the effect of the flattening plays a more significant role and the decrease of incident solar radiation is correspondingly greater. Furthermore, it is particularly evident from Figure 1 that the curves of constant ratio I_{DO}/I_D roughly parallel the Arctic Circle bounding the polar region in which there are days without sunrise.

Finally, we also have studied the latitudinal variation of the percentage difference of the mean daily insolations. It is found that for latitudes equatorward of the subsolar point, the mean summer daily insolation of an oblate Mars is increased when compared to a spherical one, the maximum rise being extremely small ($\simeq 0.05\%$). At higher latitudes, there is a loss of insolation which is of most importance at mid-latitude regions ($\simeq 0.2\%$).

In winter, the horizon plane is always tilted away from the Sun causing both the cosine of the zenith angle and the length of the day to be reduced. Consequently, the daily insolation as well as the mean daily insolation are reduced, the latter decreasing maximally by about 2% at high latitudes.

Despite of the partial gain of the mean summertime insolation near the equator, the effect of the flattening can clearly be seen to reduce the mean daily insolation over the entire year.

In conclusion, we believe that the effect of the oblateness, although very small except in the vicinity of the polar night, has to be taken into account in studies related to theoretical models for the calculation of solar global insolation on Mars.

7. REFERENCES

1. Van Hemelrijck E 1982, The oblateness effect on the solar radiation incident at the top of the atmospheres of the outer planets, Icarus (accepted).
2. Van Hemelrijck E 1982, The oblateness effect on the extraterrestrial solar radiation, Solar Energy (accepted).
3. Ward W R 1974, Climatic variations on Mars. 1. Astronomical theory of insolation, J. Geophys. Res. 79, 3375-3386.
4. Vorob'yev V I and Monin A S 1975, Upper-boundary insolation of the atmospheres of the planets of the solar systems, Atmos. Oceanic Phys. 11, 557-560.
5. Levine J S, Kraemer D R and Kuhn W R 1977, Solar radiation incident on Mars and the outer planets : Latitudinal, seasonal, and atmospheric effects, Icarus 31, 136-145.
6. Brinkman A W and Mc Gregor J 1979, The effect of the ring system on the solar radiation reaching the top of Saturn's atmosphere : Direct radiation, Icarus 38, 479-482.
7. Van Hemelrijck E and Vercheval J 1981, Some aspects of the solar radiation incident at the top of the atmospheres of Mercury and Venus, Icarus 48, 167-179.
8. Van Hemelrijck E 1982, The insolation at Pluto, Icarus (accepted).
9. Thekaekara M P 1973, Solar energy outside the Earth's atmosphere, Solar Energy 14, 109-127.
10. Goodman N J (Ed) 1980, The handbook of the British Astronomical Association 1981, Eastbourne, East Sussex, Sumfield and Day Ltd, 100-101.